Problem 1: The Earth's Horizontal Magnetic Field

Section I

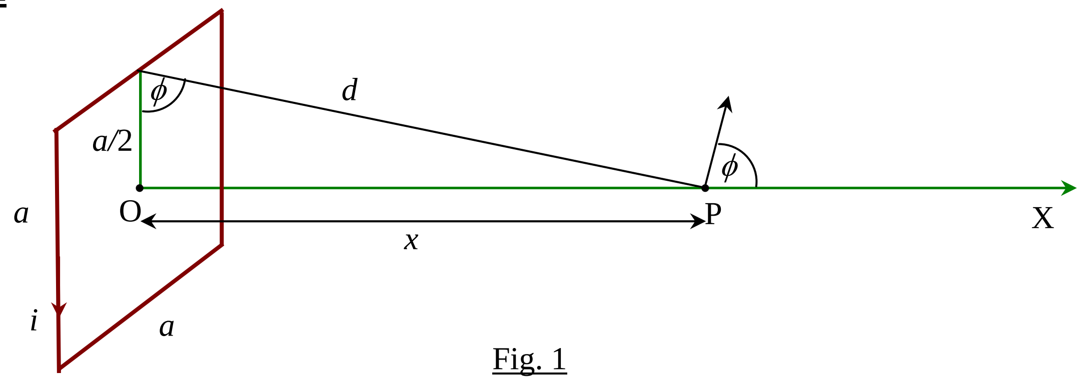


Fig. 1

At O, the centre of coil, the magnetic field for a single turn is

$$B_O = 4\times\frac{\mu_0 i}{2\pi\left(\frac{a}{2}\right)}\frac{(a/2)}{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2}} = \frac{2\sqrt{2}\mu_0 i}{\pi a}.$$

At P, the horizontal magnetic field is

$$B_{PX} = 4\frac{\mu_0 i}{2\pi d}\frac{(a/2)}{\sqrt{d^2+\left(\frac{a}{2}\right)^2}}\cos\phi.$$ [0.3 point]

From Fig. 1 we have $d = \sqrt{x^2+\left(\frac{a}{2}\right)^2}$ and $\cos\phi = \frac{(a/2)}{\sqrt{x^2+\left(\frac{a}{2}\right)^2}}$. [0.2 point]

Then, for a square coil of N turns [0.2 point]

$$B_{px} = \left(\frac{2\mu_0 iN}{\pi}\right)\cdot\frac{a/2}{\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\cdot\frac{a/2}{\left(x^2+\left(\frac{a}{2}\right)^2\right)}$$

or

$$B_{px} = \left(\frac{\mu_0 a^2 iN}{2\pi}\right)\left[\frac{1}{\left(x^2+\left(\frac{a}{2}\right)^2\right)\sqrt{x^2+2\left(\frac{a}{2}\right)^2}}\right]$$ [0.3 point]

which becomes $B_{\mathrm{O}} = \dfrac{2\sqrt{2}\mu_0 iN}{\pi a}$ as $x = 0$.

Section II

Measurements to justify that we can ignore the torsion of the string.

length of string (cm)	time for 10 oscillations (sec)
2	9.38
4	9.69
6	9.90
8	10.13
10	10.13
12	10.22
14	10.12
25	10.12

(Note that this data is from a different magnet used in Section III.)

We can see that the period is constant for length of string ≥ 10 cm.

Section III

The distance between the center of the magnet and the top surface of the platform for Part a), b) and c) is $14.0 \pm 0.5\,\mathrm{cm}$.

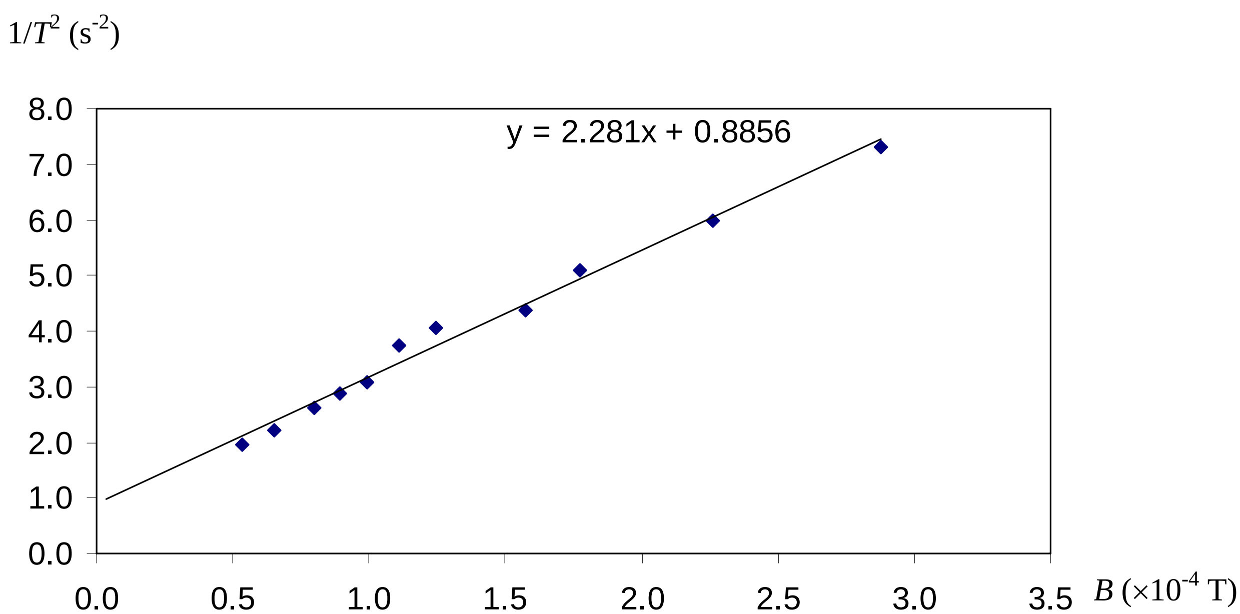
a) Coil's magnetic field and Earth's horizontal magnetic field are in the same direction

Since the coil's magnetic field (B) and Earth's magnetic field (B_{H}) are in the same direction, from

$T = 2\pi\sqrt{\dfrac{I}{mB}}$ we have $\dfrac{1}{T^2} = \beta B + \beta B_{\mathrm{H}}$ where $\beta = \dfrac{m}{4\pi^2 I}$ By plotting linear graph of $\dfrac{1}{T^2}$ and B we can find B_{H} from its slope and intercept.

Measurement of 20 oscillations at different distances from coil, we get the result as in table.

x (cm)	time for 20 oscillation (sec)		period T (sec)	$B\left(\times10^{-4}\text{T}\right)$	$1/T^2$
10	7.44	7.35	0.370	2.878	7.305
12	8.19	8.13	0.408	2.259	5.998
14	8.87	8.91	0.443	1.773	5.088
15	9.5	9.62	0.478	1.573	4.377
17	9.91	9.97	0.497	1.245	4.048
18	10.43	10.35	0.518	1.111	3.734
19	11.47	11.31	0.569	0.994	3.085
20	11.78	11.81	0.591	0.891	2.866
21	12.41	12.34	0.619	0.801	2.613
23	13.41	13.4	0.671	0.652	2.222
25	14.22	14.28	0.714	0.535	1.964



From graph we have: slope $\beta = (2.281 \pm 0.063)\times 10^4$ s^{-2}/T

intercept $\beta B_{\text{H}} = 0.886 \pm 0.076$ s^{-2}

The value of Earth's magnetic field is

$$B_{\text{H}} = \frac{0.8856}{2.281\times10^4} = 0.39\times10^{-4}\ \text{T} = 0.39\pm0.04\ \text{G}$$

The magnetic moment of magnet is $m = \beta^2 4\pi^2 M\left(\frac{L^2}{12}+\frac{r^2}{4}\right) = 1.68 \pm 0.09\ \text{A m}^2$

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b) Earth's magnetic field only

Time for 30 oscillations: 36.28, 36.25, 36.24 s.

Averaged period $T_E \quad = \quad 1.209 \pm 0.001\ \text{s}$

$$B_{\mathrm{H}} \quad = \quad \frac{1}{T_E{}^2 \beta} = \frac{1}{1.21^2 \times 2.281 \times 10^{-4}} = 0.30 \pm 0.01\,\mathrm{G}$$

c) Coil's magnetic field and Earth's horizontal magnetic field are in opposite directions

The equilibrium (neutral) position $x_0 = 31.0 \pm 0.2\,\text{cm}$.

$$B_{\mathrm{H}} = \left(\frac{\mu_0 a^2 i N}{2\pi}\right)\left[\frac{1}{\left(x_0{}^2 + \left(\frac{a}{2}\right)^2\right)\sqrt{x_0{}^2 + 2\left(\frac{a}{2}\right)^2}}\right] = 0.31 \pm 0.01\,\mathrm{G}$$

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Problem2: Oscillation of Water-Filled Vessel

Section I

i) $m_1 = \rho\pi\left[R^2-(R-t)^2\right]L = \rho\pi\left(2Rt-t^2\right)L$ ………………….. (i)

ii) $m_2 = \rho\pi\left(0.6\,\text{cm}\right)R^2$ ………………….. (ii)

iii) $m_3 = \pi\left(R-t\right)^2 L$ ………………….. (iii)

iv) $M = m_1+2m_2+m_3$ ………………….. (iv)

v) Water, as an **ideal** fluid, does not take part in the oscillatory motion of the water-filled vessel. We therefore shall not include contribution of water in the expressions for the moments of inertia.

$$I_y = \frac{1}{2}m_1\left[R^2+(R-t)^2\right]+2\left[\frac{1}{2}m_2R^2\right] \qquad \text{………………….. (v)}$$

$\ell = 35.6$ cm, $g = 978$ cm s^{-2}, $a = R = 2.5$ cm,

$\rho = 2.70$ g cm^{-3}, $h = 9.2$ cm, $L = 9.2-1.2 = 8.0$ cm

$$m_1 = 339.3t-67.86t^2 \quad \text{g},$$

$$m_2 = 31.8 \quad \text{g},$$

$$m_3 = 157.1-125.7t+25.13t^2 \quad \text{g},$$

$$M = 220.7+213.6t-42.73t^2 \quad \text{g}$$

$$I_y = 198.8+2121t-1273t^2+339.3t^3-33.93t^4$$

Section II

a)

$$T_y = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_y}{Ma^2}} \qquad \text{………………….. (vi)}$$

Time for 50 oscillations is 43.3 s, hence

$$T_y = 0.866\ \text{s} \pm 0.004\ \text{s}$$

Substituting these values into equation (vi), we get

$$t^4-10t^3+33.42t^2-41.97t+15.36 = 0 \qquad \text{………………….. (vii)}$$

The solution of (vii), by numerical iteration, is $t = 0.62 \text{ cm} \pm 0.02 \text{ cm}$

(The error can be estimated from a repeat of the procedure or a differential equation for dT and dt)

Hence, we get

$$m_1 = 184 \pm 5 \text{ g},$$

$$m_2 = 31.8 \pm 0.2 \text{ g},$$

$$m_3 = 89 \pm 2 \text{ g},$$

$$M = 337 \pm 6 \text{ g}.$$

b)

$$T_x = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_x}{Ma^2}} \qquad \text{………………….. (viii)}$$

$\ell = 33.6 \text{ cm}, \quad a = \frac{h}{2} = \frac{9.2}{2} = 4.6 \text{ cm}$

Time for 50 oscillations is 38.0 s $\pm$ 0.2 s, hence

$$T_x = 0.760 \text{ s} \pm 0.004 \text{ s}$$

And from $T_x = 2\pi\sqrt{\frac{\ell}{g}\cdot\frac{I_x}{Ma^2}}$ where $\ell = 33.6 \text{ cm}, \quad a = \frac{h}{2} = \frac{9.2}{2} = 4.6 \text{ cm}$,

$g = 978 \text{ cm s}^{-2}$, $M = 337 \text{ g}$, we get

$$I_x^{\text{Exp}} = 3036 \text{ g cm}^2 \pm 94 \text{ g cm}^2$$

Also $I_x^{\text{Theo}} = 3261 \text{ g cm}^2 \pm 68 \text{ g cm}^2$

c)

$$\Delta I_x = I_x^{\text{Theo}} - I_x^{\text{Exp}} = 225 \text{ g cm}^2$$

The experimental value I_x^{Exp} is smaller than I_x^{Theo} by 225 g cm^2.

This difference is probably significant and it is due to low viscosity of water. The mass of water in the middle section does not take part in the oscillatory motion of the vessel.

ΔI_x can be estimated to be due to a stationary cylindrical portion of water in the middle.

$$\Delta I_x = \pi(R-t)^2 L_{water}\left(\frac{L_{water}{}^2}{12}+\frac{(R-t)^2}{4}\right)$$

$L_{water} \approx 5.7\,\text{cm}$

This corresponds to the water mass of $\approx 63.8\ g$

The percentage of the water that takes part in the oscillation is $\approx 28.5\%$

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