Experimental Question 1: Levitation of Conductors in an Oscillating Magnetic Field

In an oscillating magnetic field of sufficient strength, levitation of a metal conductor becomes possible. The levitation occurs due to a non-zero mean magnetic force exerted on currents in the conductor. The currents are induced by the alternating field itself. The Lorentz force doesn't average to zero because of a phase shift between the current oscillations and the magnetic field oscillations. This phase shift is a result of the self-inductance of the current loops within the conductor.

In this experiment, we study this phenomenon and deduce the self-inductance of an aluminum ring from measuring the vertical force applied to it by a solenoid with an oscillating current. For measurement convenience, the mean force on the ring will be directed downwards, so levitation will not be observed.

On your desk, you have the following items (Figure 1):

(1) An AC power supply operating on 50Hz. The power supply has two pairs of terminals. The two smaller sockets (1a) supply a voltage of about 24V; use these only for running current through the solenoid. The two larger sockets (1b) supply a voltage of about 0.7V. The power supply is turned on only when the green button (1c) is pressed – this is in order to prevent overheating the system by accidentally leaving the current on, as further explained below. The red light bulb indicates when the power supply is on.

(2) A cylindrically symmetric solenoid filled with iron rods. The solenoid is connected to the 24V terminals of the power supply. The solenoid can be raised and lowered using a long screw. The screw's vertical step is $h = 1.41mm$.

(3-5) Three metal rings made of exactly the same material (an alloy of aluminum). One ring is closed. The second ring is identical to the first except for a short segment which was removed, making the ring open. The third ring is also open, and is much thinner than the first two.

(6-7) Two multimeters. They will be used as a voltmeter and an ammeter. The AC voltmeter's sensitivity is 0.1mV. The ammeter can measure currents of up to 20A (also in AC mode). **Note:** in AC mode, the multimeters display the RMS (root-mean-square) of the measured quantity, i.e. the amplitude divided by $\sqrt{2}$. See Figures 3 and 4 for detailed instructions.

(8) A battery-powered digital scale with sensitivity 0.01g. When the scale experiences a rapidly oscillating force, it displays the time-averaged force. **Note:** the scale has a "tare" option, which calibrates its reading under a given weight to zero. See Figure 2.

(9) An 8cm×7cm×7cm polystyrene block which can be used as a stand for the rings.

(10) Electric wires with various connectors.

(11) A ruler.

(12) Millimeter graph paper.

(13) A desktop lamp which can be turned on or off for your convenience.

Caution: when the closed ring is exposed to the solenoid's magnetic field, a large current flows through it, heating it up. As a consequence, the ring's electrical properties may change slightly. To avoid this, don't run a current through the solenoid for long periods of time.

The earth's gravity field in Tel Aviv is $g = 9.80 \pm 0.01\text{N/kg}$.

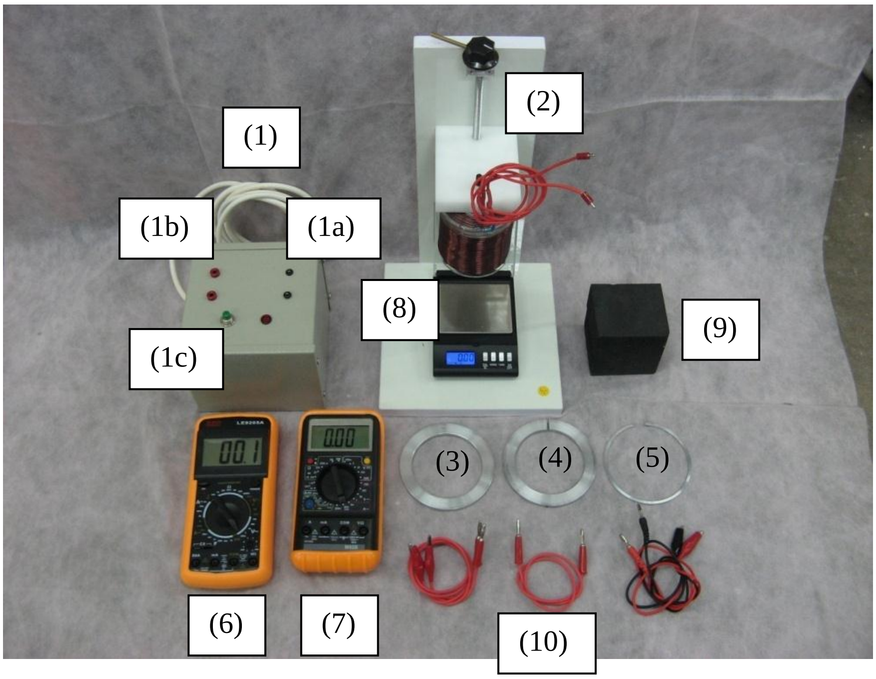

Figure 1 - Summary of the equipment.

(1) 50Hz power supply.
 (1a) 24V terminals.
 (1b) 0.7V terminals.
 (1c) Power button.
(2) Solenoid on a vertical screw.
(3) Broad closed ring.
(4) Broad open ring.
(5) Thin open ring.
(6) Voltmeter.
(7) Ammeter.
(8) Scale.
(9) Polystyrene block.
(10) Wires.

Figure 2: The digital scale

(1) On/Off button
(2) “Tare” button – sets the current weight as 0.

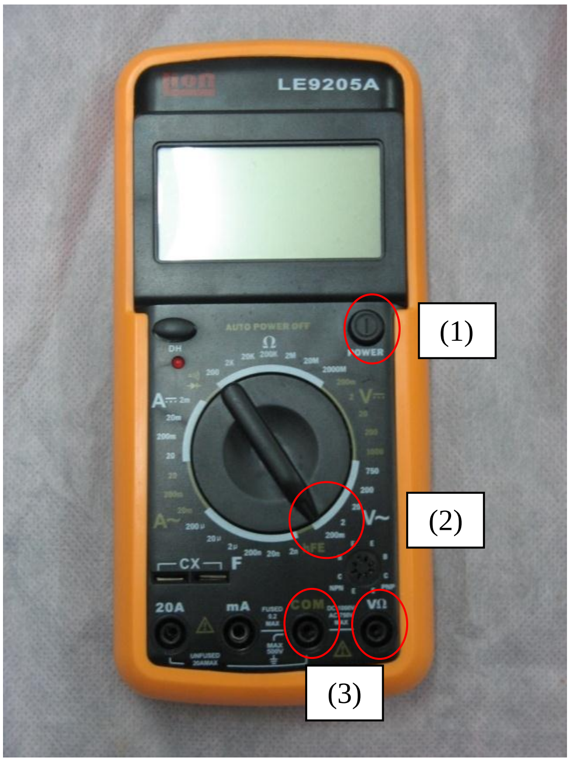

Figure 3: The voltmeter.

(1) On/Off button.
(2) The dial is set to 200mV AC.
(3) Connect your wires to the “COM” and “V/Ω” terminals.

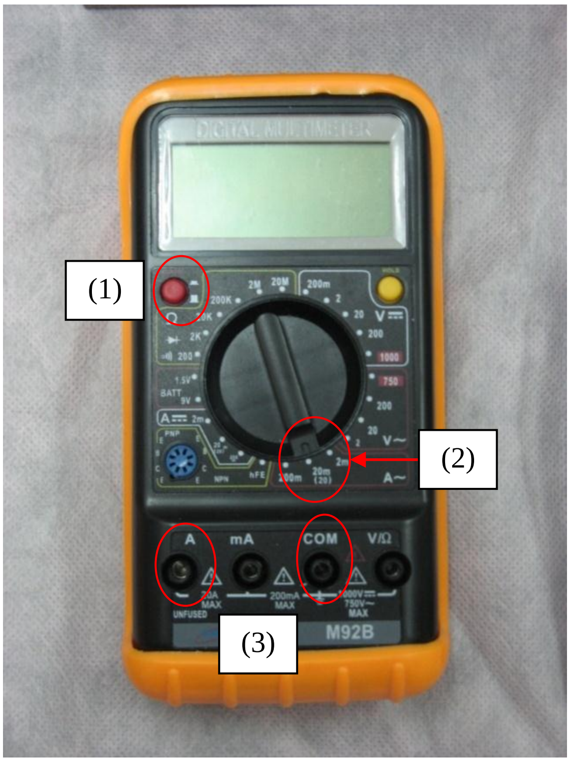

Figure 4: The ammeter.

(1) On/Off button.
(2) The dial is set to 20mA/20A AC.
(3) Connect your wires to the “COM” and “A” terminals.

Theory (1.3 points)

Consider a conducting ring of radius r placed in the solenoid's magnetic field. The symmetry axes of the solenoid and the ring coincide. Denote the ring's inductance by L, its resistance by R, and the angular frequency of the solenoid's current by ω. Define z as the coordinate along the common symmetry axis of the solenoid and the ring.

In this part only, you may neglect the small effect of the ring's magnetic field on the solenoid and the iron. Also, neglect the thickness of the ring.

In this part, we'll use Faraday's law, and the magnetic version of Gauss's law:

- Faraday's law: The induced EMF (electromotive force) on a loop generated by a changing magnetic flux is $\epsilon = -d\Phi_{\mathrm{B}}/dt$.
- The magnetic Gauss law: the total magnetic flux through a closed surface is zero.

A current loop placed in a cylindrically symmetric magnetic field $\vec{B}$ experiences a total force

$$F(t) = -2\pi r I(t) B_r(t)$$

where I is the current in the loop, and B_r is the radial component (in the direction of the loop's radius) of the external magnetic field in the loop's vicinity. The positive direction of the force F is downwards – in the z direction. The positive direction of the current I is shown in Figure 5.

a. (0.2 pts.) Consider an oscillating external magnetic flux $\Phi_{\mathrm{B}}(t) = \sqrt{2}\Phi_{\mathrm{B}}^{\mathrm{rms}} \sin(\omega t)$ through the ring.
Find $\epsilon(t)$ - the EMF induced by the given flux only, and $I(t)$ - the current induced in the ring, as functions of $\Phi_{\mathrm{B}}^{\mathrm{rms}}$, L, R, ω and t.
Hint: The EMF amplitude ϵ_0 and the current amplitude I_0on an AC circuit element with both resistance and inductance are related by $\epsilon_0 = I_0\sqrt{\omega^2 L^2 + R^2}$, and the current is delayed by a phase $\delta = \tan^{-1}\frac{\omega L}{R}$ with respect to the EMF.

b. (0.6 pts.) Find B_r in terms of r and $\frac{\mathrm{d}}{\mathrm{dz}}\Phi_{\mathrm{B}}$, where z is the coordinate along the axis perpendicular to the ring's plane.

c. (0.5 pts.) Show that $\langle F \rangle = \alpha \frac{L}{(R^2+\omega^2 L^2)} \cdot \frac{d(\epsilon_{rms})^2}{dz}$, where $\langle F \rangle$ is the time averaged value of F , $\epsilon^{rms}(z)$ is the RMS (root-mean-square, i.e., amplitude divided by $\sqrt{2}$) of the EMF on a loop at height z. Find the constant α (if you do not find α, in later parts, take the magnitude of α to be 1).
Hint: you may find the following identities useful:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$\langle(\sin(\omega t))^2\rangle = \langle(\cos(\omega t))^2\rangle = \frac{1}{2}$$

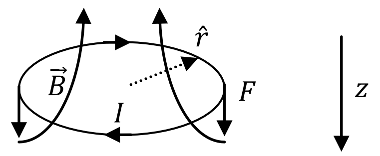

Figure 5: The metal ring in the solenoid's magnetic field.

Measurements (5.1 points)

In all of the following measurements and analysis, take into account that results with higher precision will receive higher grades. In all of your measurements and results, specify error estimations.

Resistance measurements (2.6 points)

In this part, you should use the 0.7V terminals of the AC power supply. Using the supplied wires to short-circuit the two 0.7V terminals should result in a current of $5A - 15A$, depending on the contacts. Note that the three shorter wires achieve a better contact with the ammeter than the two longer ones. CAUTION: Don't use the 24V terminals, to avoid overheating the components.

d. (1.3 pts.) Find the resistance R_{thin} of the thin ring. Draw your circuit on the answer form.
Hint: the resistance of each of the rings is much smaller than 0.1Ω. *For the thin ring, you can neglect the inductive impedance with respect to the resistance.*

e. (1.3 pts.) Find the resistance R of the closed ring. Make additional measurements as necessary.

Measurements of the induced EMF (1.5 points)

f. (1.5 pts.) Connect the solenoid to the 24V terminals of the power supply. Place the broad open ring so that its axis coincides with the axis of the solenoid. Measure the induced EMF ϵ_{rms} on the ring at different heights z, i.e., at different distances from the solenoid. Record your measurements in the provided table on the answer form. Plot a graph of ϵ_{rms} as a function of z (with a trend line).

Measurements of the force (1 point)

g. (1 pt.) Place the broad closed ring so that its axis coincides with the axis of the solenoid. Measure the time-averaged magnetic force $\langle F \rangle$ on the ring at different heights z, i.e. at different distances from the solenoid. Record your measurements in the provided table on the answer form.

Analysis (3.6 points)

h. (1.4 pts.) Find the absolute value of the derivative $|d\epsilon_{rms}^2/dz|$ of ϵ_{rms}^2 with respect to z, for values of z where you measured the force in part (g). Record your values in the provided table on the answer form. Error estimations are not required in this part.

i. (2.2 pts.) Analyze your results using a linear graph to find L - the inductance of the closed ring. You may use the fact that $\omega L < R$.
Note: Despite the noticeable thickness of the closed ring, the formula you derived in part (c) still applies with a high accuracy. Use it as an operational *definition* for the inductance of a broad ring.
Hint: When the ring is too close to the iron, the measurements will be distorted. Try to avoid this complication in your analysis.

Experimental Question 2: An Optical "Black Box"

TV and computer screens have advanced significantly in recent years. Today, most displays consist of a color LCD filter matrix and a uniform white backlight source. In this experiment, we will study a sample of plastic material which was considered for use as an ingredient in the backlight illumination of LCD screens.

Equipment

On your desk, you have the following items (see Figure 1):

(1) The sample - a piece of plastic material fixed in a slide frame. **The sample is sensitive - do not touch it. To adjust the sample's position, use its holder and stand.**

(2) A holder and stand for the slide frame. The stand includes a handle which can be used for fine rotations of the sample. **Do not remove the slide frame from its holder.**

(3) A white LED flashlight. The flashlight can be turned on and off using a button at its rear end. **Do not confuse it with the laser (see Figure 2).**

(4) A red laser pointer. **The laser is marked with a warning label. Do not confuse it with the white flashlight (see Figure 2).** The laser may be turned on and off by moving its black cap back and forth. **Don't try to remove the cap – it may be dangerous, and you may break the laser.** The laser's battery will weaken after about an hour - do not keep it turned on longer than necessary. The laser's wavelength is $\lambda = 652\text{nm} \pm 2\text{nm}$.

(5) A single stand to be used for the two light sources. **At the start of the experiment, the flashlight is fixed to the stand, while the laser lies on the desk.**

(6) A white screen on wooden legs, covered with millimeter graph paper. There is a hole near the middle of the screen. You are allowed to make markings on the screen.

(7) A wooden stake that can be moved back and forth on a wooden bench. You are allowed to make markings on the bench.

(8) A tape measure.

(9) A ruler.

(10) Millimeter graph paper.

(11) A desktop lamp which can be turned on or off for your convenience.

LASER SAFETY:

1. **Do not stare into the laser beam!**
2. **Beware of reflections from metallic surfaces.**
3. **Do not point the laser at others.**
4. **Do not try to repair or disassemble the laser.**
   Call a supervisor if you require assistance.

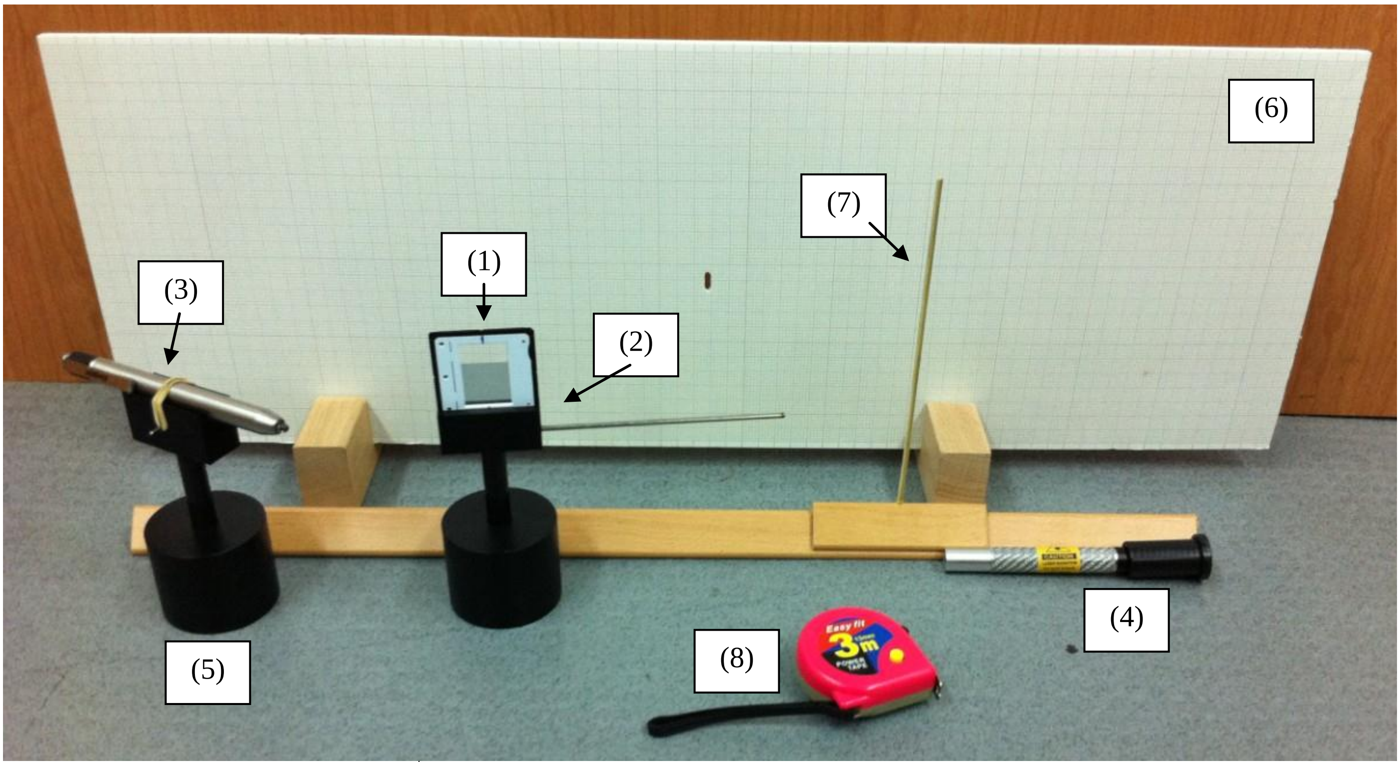

Figure 1: Summary of the equipment.

1. The plastic sample.
2. Sample holder and stand.
3. White LED flashlight.
4. Red laser.
5. Stand for light source.
6. Screen covered with millimeter graph paper.
7. Wooden stake on bench.
8. Tape measure.

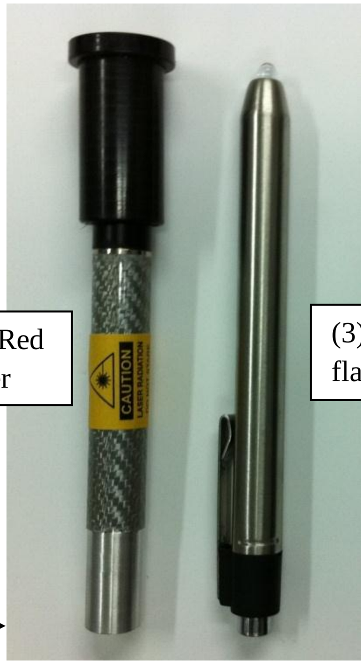

Figure 2: Close-up of the two light sources.

Part I – Theory (0.4 points)

a. (0.4 pts.) A light ray is reflected from two mirrors which meet at an angle φ (Figure 3). Find the angle γ between the incoming and outgoing rays. Assume that all light rays lie in the plane perpendicular to the mirrors' intersection line.

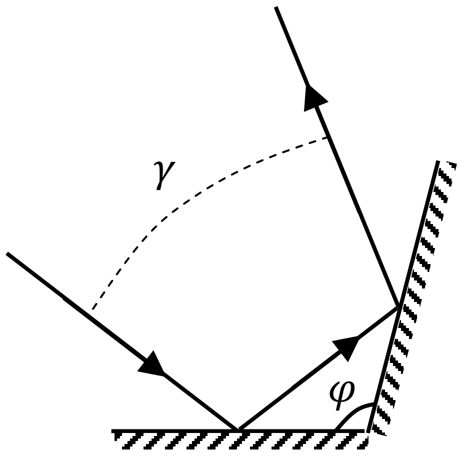

Figure 3: A light ray reflected from two mirrors.

Part II – Measurements with white light (6.1 points)

Using the white flashlight as your light source, you may observe both the transmission and the reflection properties of the sample. Figure 4 illustrates the suggested setups for both types of observation. Note: you may observe different results when illuminating the two sides of the sample.

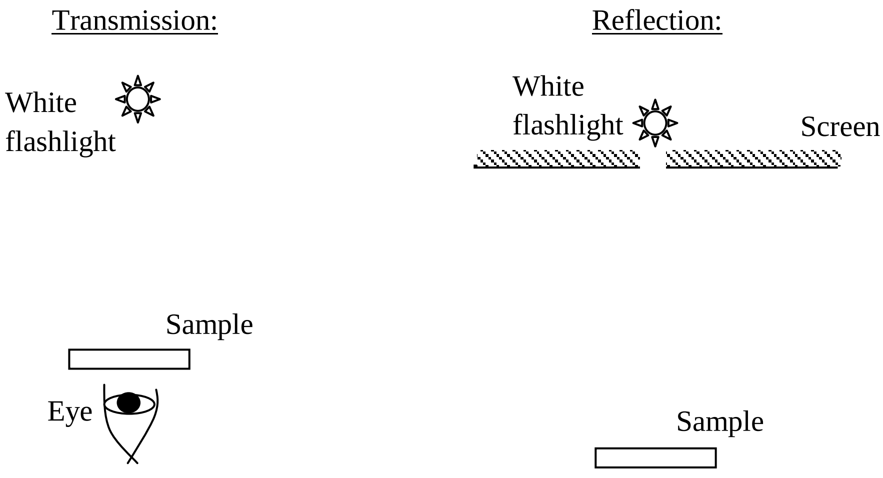

Figure 4: Suggested observation setups for white light.

CAUTION: For viewing transmitted light, you will have to look directly into the flashlight beam through the sample. **Don't do this with the laser!** Also, avoid looking directly into the flashlight itself for long periods of time.

b. (0.5 pts.) Figure 5 illustrates schematically four possibilities for the sample's microscopic structure. n stands for the refractive index of the plastic. Choose the structure that best fits your observations. Note: the 5 periods shown in the figure are for illustration only. In reality, d is small, and the sample contains many periods.

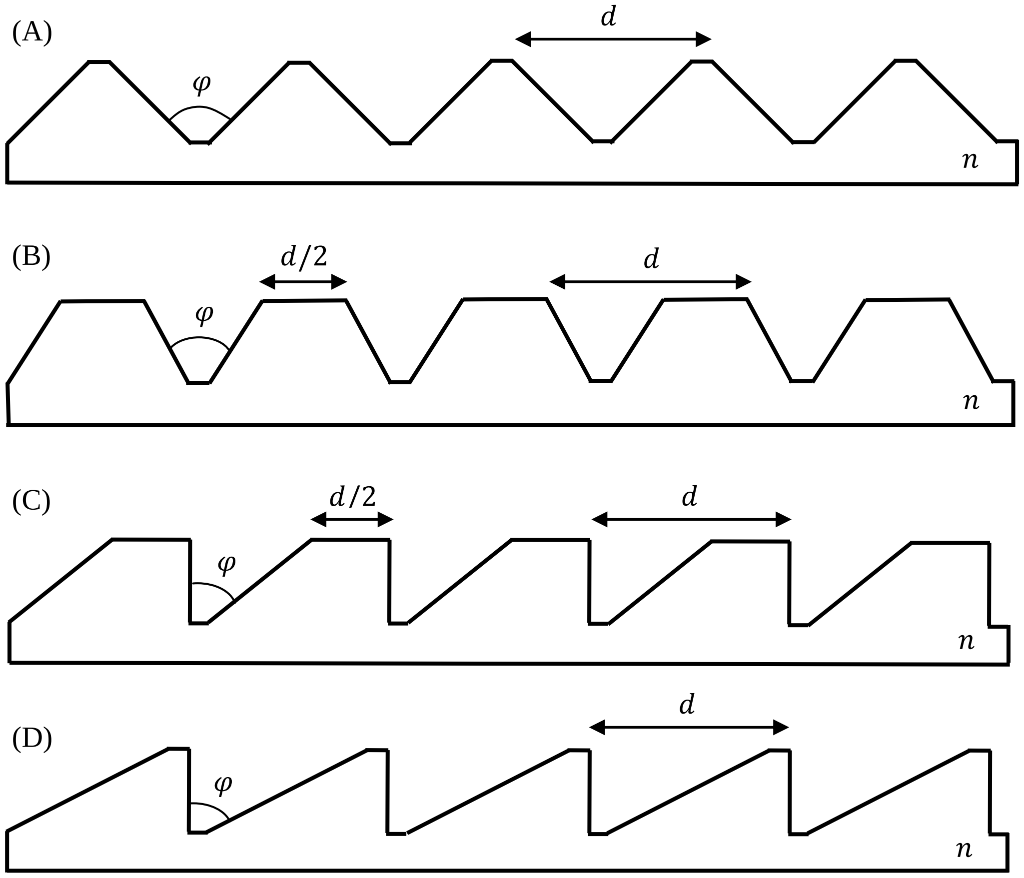

Figure 5: Different possibilities for the sample's structure.

c. (0.8 pts.) Find the angle φ for the sample and estimate its error.

d. (0.5 pts.) When a perpendicular white light beam is incident on the sample from one of its sides, the following faint pattern can be observed in the transmitted light, slightly to the right from the source (Figure 6). "R", "G" and "B" stand for red, green and blue respectively. Note: this pattern may be difficult to observe, and measurements on it are not required.

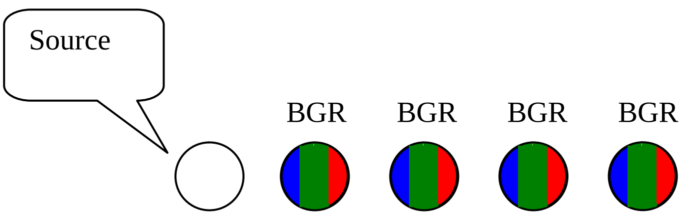

Figure 6: Faint pattern near the light source.

Further to the right you may observe a much brighter pattern (Figure 7):

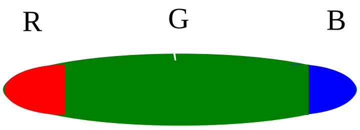

Figure 7: Bright pattern farther to the right from the light source.

Choose the correct option:

A. All the colored patterns result from interference.
B. All the colored patterns result from the dependence of n on the wavelength.
C. The patterns depicted in Figure 6 result from interference, while the pattern depicted in Figure 7 results from dependence of n on the wavelength.
D. The patterns depicted in Figure 6 result from the dependence of n on the wavelength, while the pattern depicted in Figure 7 results from interference.

e. (1.4 pts.) With the white light set up as in part (d), measure the deflection angle δ_0 of violet light (at the far blue end of the spectrum) for the dominant peak depicted in Figure 7. The deflection angle is defined in Figure 8. Record all intermediate measurements. Provide error estimates.

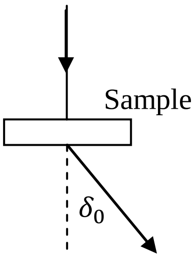

Figure 8: The deflection angle δ_0.

f. (1.4 pts.) Illumination of the sample at different angles of incidence results in different deflection angles for the dominant transmitted peaks. Measure the minimal deflection angle δ_{min} of the dominant peak for transmitted violet light (there is only one such minimal angle). Record all intermediate measurements. Provide error estimates.

g. (0.8 pts.) Using the angle φ from part (c), express the refraction index n of the sample in terms of either δ_0 or δ_{min}. You may use the reversibility of light propagation and the fact that there is only one minimal angle δ_{min}.

h. (0.7 pts.) Find the refraction index n_v of the sample for violet light and its error estimate.

Part III - Laser measurements (3.5 points)

Remove the flashlight from the light-source stand, and replace it with the laser. You can use the white screen to view both transmission and reflection patterns, as illustrated in Figure 9. The laser has a limited battery life - do not keep it turned on longer than necessary. When aligning the components, it may help to rotate the laser around its axis.
WARNING: Do not look directly into the laser beam or its reflections! Do not look at the laser light through the sample – use the provided screen.

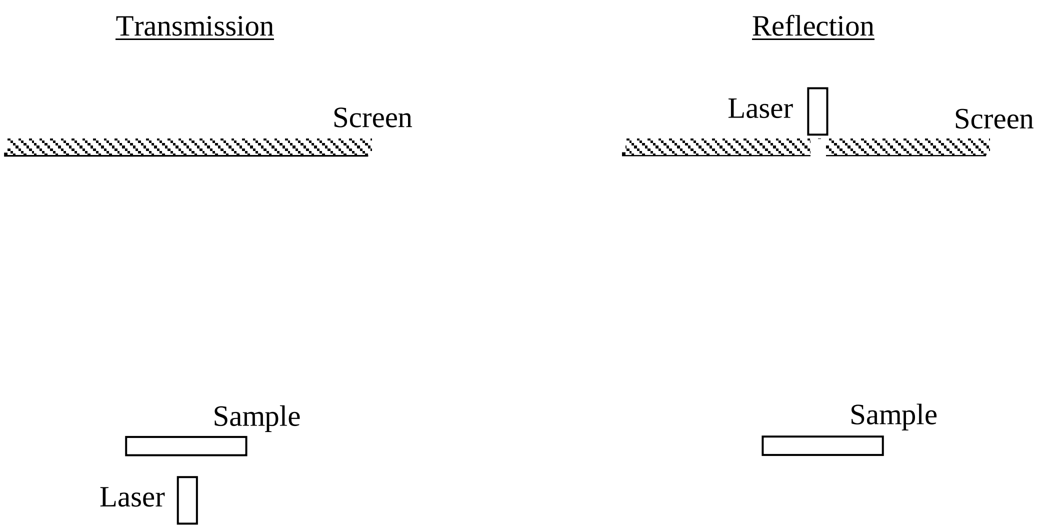

Figure 9: Suggested observation setups for laser light.

Observe the alternating pattern of bright and dim fringes on the screen as you slightly rotate the sample. The dimming of some of the fringes is due to destructive interference between different regions of each "tooth" on the sample.

i. (1 pt.) Use one of the setups in Figure 9, with the sample illuminated perpendicularly by the laser beam. Record the deflection angles θ of the observed fringes as a function of the fringe number m. Define the center of the pattern as $m = 0$. Use the provided table on the answer form. Record all intermediate measurements. Provide error estimations.

j. (1.5 pts.) Using a linear graph, find the spacing d between two adjacent "teeth" of the sample. Error bars on the graph are not required. Provide error estimation for d.

k. (1 pt.) Using the formula you derived in part (g), find the refraction index n_r of the sample for the laser's red wavelength. Record any additional measurements. Provide error estimates. **WARNING: Do not look through the sample! Use the provided screen.**