Model Solution

We will assume the relationship of the form:

$$P = f(W)\, h(\theta)$$

$$M_p = f(M_w)\, h(\theta)$$

M_w represents the mass in the hanger (Load)
M_p represents the mass in the pan + the mass of the pan (i.e. $M'_p + M_{pan}$) (Effort)

The relation between these variables can be found in two parts:

- Relation between M_P and M_W
- Relation between M_P and θ

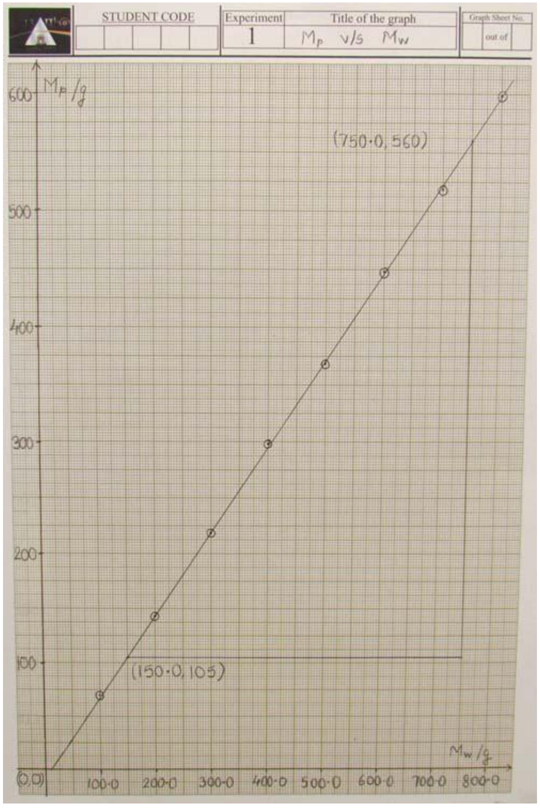
Part 1:

Mass of the pan = 28.6 g

$\theta = \pi$ radians

Obs. No.	M_w/g	M'_{p-} /g	M'_{p+} /g	$M'_{p(average)}$ /g	$M_{p(average)} = M'_{p(average)} + M_{pan}$ /g	$\Delta M_p = \frac{M'_{p+} - M'_{p-}}{2}$ /g
1	800.0	566	574	570	598.6	4
2	700.0	486	494	490	518.6	4
3	600.0	417	423	420	448.6	3
4	500.0	337	343	340	368.6	3
5	400.0	267	273	270	298.6	3
6	300.0	188	192	190	218.6	2
7	200.0	113	117	115	143.6	2
8	100.0	41	43	42	70.6	1

Graph of M_P v/s M_W:



Slope of the graph = 0.7583

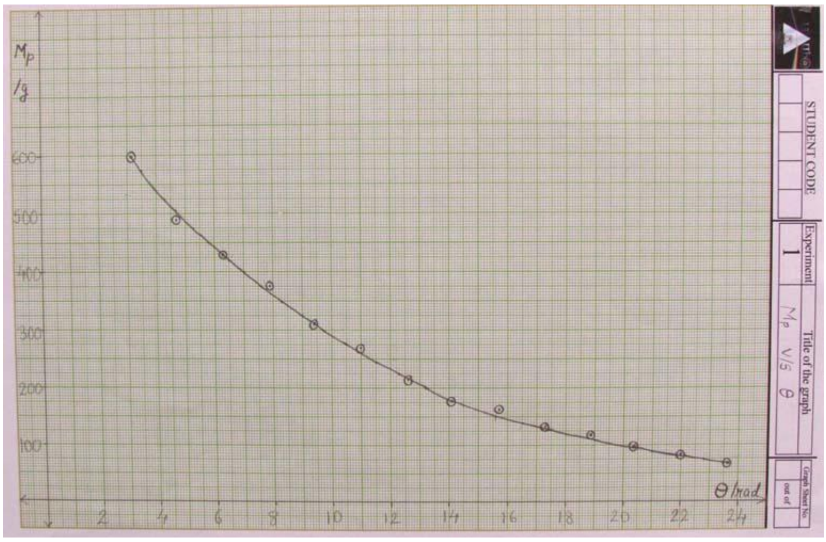
This shows that

$$P \propto W \tag{1}$$

Part 1:

M_w = 800.0 g
M_{pan} = 28.6 g

Obs. No.	θ /rad	M'_{p-} /g	M'_{p+} /g	$M'_{p(average)}$ /g	$M_{p(average)} = M'_{p(average)} + M_{pan}$ /g	$\Delta M_p = \frac{M'_{p+} - M'_{p-}}{2}$ /g
1	π	565	575	570	598.6	5
2	$3\pi/2$	455	465	460	488.6	5
3	2π	396	404	400	428.6	4
4	$5\pi/2$	316	324	320	348.6	4
5	3π	276	284	280	308.6	4
6	$7\pi/2$	236	244	240	268.6	4
7	4π	182	188	185	213.6	3
8	$9\pi/2$	147	153	150	178.6	3
9	5π	133	137	135	163.6	2
10	$11\pi/2$	104	110	107	135.6	3
11	6π	88	92	90	118.6	2
12	$13\pi/2$	68	72	70	98.6	2
13	7π	54	56	55	83.6	1
14	$15\pi/2$	39	41	40	68.6	1
15	8π	29	31	30	58.6	1
16	$17\pi/2$	18	20	19	47.6	1



The graph between M_p and θ shows a curve.

There can be possibilities of different functional relationship.

1)

The possible functions can be $\frac{1}{\theta}, \frac{1}{\theta^2}, e^{-k\theta}$

For the first two functions mentioned above, at θ = 0, M_p will reach infinite value which is not possible. For the third function we know that M_p will have some finite value.

2)

If the function is anticipated as exponential one then it can be verified using half value technique whether at every half value of Mp, and then plotting ln M_p or better still $\ln\left(M_p/M_w\right)$ against θ.

If it is a straight line with slope $-k$,

$$M_p \propto e^{-k\theta}$$

or

$$P \propto e^{-k\theta} \tag{2}$$

From (1) and (2),

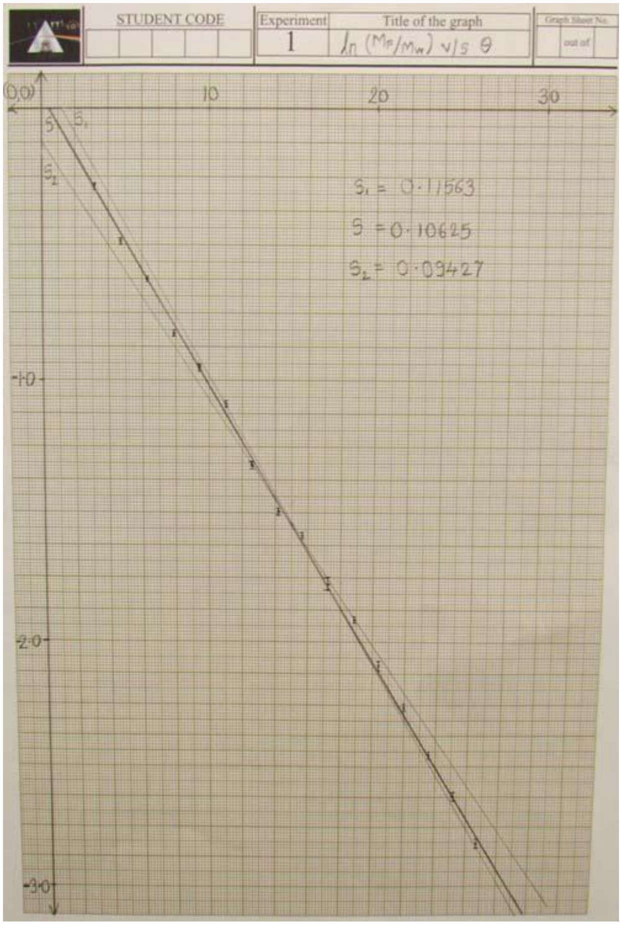
$$P \propto We^{-k\theta} \tag{3}$$

The constant k in the above expression is equated to the coefficient of friction, μ.

$$P = CWe^{-\mu\theta} \tag{4}$$

The constant of proportionality in equation (4) is 1. This is because at $\theta = 0$ rad, $P = W$.

$$P = We^{-\mu\theta} \tag{5}$$



From the graph,

$$\ln\left(\frac{M_p}{M_w}\right) = -\mu\theta$$

where μ is the slope of the graph.

From the graph, μ = 0.106

$$\frac{\Delta S}{S} = \frac{(S_1 \sim S_2)/2}{S} = 0.10049$$

$$\Delta S = 0.01067 = \Delta\mu$$

$$u_C(\mu) = 0.00616$$

$$U(\mu) = 0.0123 \approx 0.02$$

$$\mu = 0.11 \pm 0.02$$

Part 2:

When the pan is moving up:

$$M_{p1} = M_u e^{-\mu\theta} \tag{6}$$

When the pan is moving down:

$$M_{p2} = M_u e^{\mu\theta} \tag{7}$$

M'_{p1}		$M'_{p1(average)}$	$\Delta M'_{p1}$	$M_{p1} = M'_{p1} + M_{pan}$
M'_{p1+}	M'_{p1-}			
164	156	160	4	188.6

M'_{p2}		$M'_{p2(average)}$	$\Delta M'_{p2}$	$M_{p2} = M'_{p2} + M_{pan}$
M'_{p2+}	M'_{p2-}			
49	45	47	2	75.6

For M_u:

Multiplying equation (6) and (7),

$$M_u = \sqrt{M_{p1} \cdot M_{p2}}$$

$$M_u = \sqrt{188.6 \times 75.6} = 119.4075g$$

For μ:

Dividing equation (6) by (7),

$$\frac{M_{p1}}{M_{p2}} = e^{2\mu\theta}$$

$$\mu = \frac{1}{2\theta} \ln\left(\frac{M_{p1}}{M_{p2}}\right)$$

For $\theta = \pi$

$$\mu = \frac{1}{2\pi} \ln\left(\frac{M_{p1}}{M_{p2}}\right)$$

$$\mu = \frac{1}{2\pi}\ln\left(\frac{188.6}{75.6}\right) = 0.1456$$

Uncertainty in M_u:

$$\frac{\Delta M_u}{M_u} = \sqrt{\left(\frac{1}{2}\frac{\Delta M_{p1}}{M_{p1}}\right)^2 + \left(\frac{1}{2}\frac{\Delta M_{p2}}{M_{p2}}\right)^2} = \sqrt{\left(\frac{1}{2}\frac{4}{188.6}\right)^2 + \left(\frac{1}{2}\frac{2}{75.6}\right)^2} = 0.0169$$

$$\Delta M_u = 0.0169 \times 119.4075 = 2.018$$

$$u_C(M_u) = \frac{1}{\sqrt{3}} \times 2.018 = 1.165$$

$$U(M_u) = 2 \times u_C(M_u) = 2 \times 1.165 = 2.33 \approx 3$$

$$M_u = 119 \pm 3\,\mathrm{g}$$

Uncertainty in μ_u:

$$u_C(\mu_u) = \frac{1}{\sqrt{3}} \cdot \Delta\mu_u = \frac{1}{2\pi\sqrt{3}}\sqrt{\left(\frac{\Delta M_{p1}}{M_{p1}}\right)^2 + \left(\frac{\Delta M_{p2}}{M_{p2}}\right)^2} = \frac{1}{2\pi\sqrt{3}}\sqrt{\left(\frac{4}{188.6}\right)^2 + \left(\frac{2}{75.6}\right)^2} = 0.003117$$

$$U(\mu_u) = 2 \times u_C(\mu_u) = 2 \times 0.003117 = 0.00623 \approx 0.007$$

$$\mu_u = 0.146 \pm 0.007$$

Model Solution

Part 1(a):

				Coil Blue (1-1)				AIR CORE		f = 1.000 KHz		
	R'	V_A	$V_{R'}$	V	Vo	I_1	Z_1	R_1	X_1	ωM	M (mH)	L_1(mH)
	450	10.090	6.760	6.770	4.940							
		10.060	6.700	6.740	4.910							
Avg	450	10.075	6.730	6.755	4.925	0.015	451.67	52.57	448.60	329.31	52.44	71.43

ΔV_A	ΔVR'	ΔV
0.14	0.11	0.11

us(z)	us(R)	ur(Z)	ur(R)	uc(R)	uc(Z)	uc(X)	uc(L)	U(R)	U(X)	U(L)
0.0057	1.1432	0.2747	0.4235	1.2191	0.2748	0.3114	0.000050	3.00	0.7	0.0002

Part 1b:

	Coil Green (2-2)							AIR CORE		f = 1.000 KHz		
	R'	V_A	$V_{R'}$	V	Vo	I_2	Z_2	R_2	X_2	ωM	M (mH)	L_2(mH)
	350	9.960	6.690	6.610	6.390							
		9.980	6.630	6.680	6.410							
Avg	350	9.970	6.660	6.645	6.400	0.019	349.21	42.96	346.56	336.34	53.56	55.18

ΔV_A	ΔVR'	ΔV
0.14	0.11	0.11

us(z)	us(R)	ur(Z)	ur(R)	uc(R)	uc(Z)	uc(X)	uc(L)	U(R)	U(X)	U(L)
0.0027	0.9057	0.2153	0.3335	0.9651	0.2153	0.2478	0.000039	2.00	0.5	0.0001

Part 1(c)

				Coil Blue (1-1)				Al CORE		f = 1.000 KHz		
	R'	V_A	$V_{R'}$	V	Vo	I^*_1	Z^*_1	R^*_1	X^*_1	ωM	M* (mH)	L^*_1(mH)
	290	9.900	5.940	6.010	4.450							
		9.880	5.890	6.050	4.410							
Avg	290	9.890	5.915	6.030	4.430	0.020	295.64	109.68	274.54	217.19	34.58	43.72

ΔV_A	ΔVR'	ΔV
0.14	0.10	0.10

us(z)	us(R)	ur(Z)	ur(R)	uc(R)	uc(Z)	uc(X)	uc(L)	U(R)	U(X)	U(L)
0.0220	1.2498	0.2030	0.3740	1.3046	0.2042	0.5657	0.000090	3.00	1.2	0.0002

Part 1(d):

	Coil Green (2-2)						Al CORE		f = 1.000 KHz		
	R'	V_A	$V_{R'}$	V	Vo	I^*_2	Z^*_2	R^*_2	X^*_2	ωM	M*(mH)	L^*_2(mH)
	280	9.860	6.280	6.170	4.840							
		9.830	6.230	6.130	4.860							
Avg	280	9.845	6.255	6.150	4.850	0.022	275.30	71.48	265.86	217.11	34.57	42.33

ΔV_A	ΔVR'	ΔV
0.14	0.10	0.10

us(z)	us(R)	ur(Z)	ur(R)	uc(R)	uc(Z)	uc(X)	uc(L)	U(R)	U(X)	U(L)
0.0173537	0.9509	0.1821	0.3106	1.000	0.1829	0.329	0.000052	3.00	0.7	0.0002

Part 2(f):

$$\omega M = R' \frac{V_O}{V_{R'}}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

M_{avg} (Air) =	53.00 mH		k =	0.844
M^*_{avg} (Al core) =	34.58 mH		k^* =	0.804

Part 2(g):

						AIR Core				f = 1.000 KHz R'=300Ω	
	Sr.	R_L	V_A	$V_{R'}$	V	V_{RL}	Ip	Zp	R_{PE}	X_{PE}	Is
	1	100	10.19	5.99	4.97	1.73					
			10.15	6.03	4.94	1.72					
Avg		100	10.17	6.01	4.96	1.73	0.0200	247.34	177.56	172.19	0.017
	2	200	10.12	5.39	5.67	2.77					
			10.09	5.44	5.70	2.79					
Avg		200	10.11	5.42	5.69	2.78	0.0181	314.96	207.03	237.36	0.014
	3	300	10.13	5.17	6.17	3.44					
			10.15	5.11	6.19	3.43					

Avg		300	10.14	5.14	6.18	3.44	0.0171	360.70	216.93	288.18	0.011
	4	400	10.11	5.00	6.51	3.88					
			10.14	5.05	6.53	3.87					
Avg		400	10.13	5.03	6.52	3.88	0.0168	389.25	206.46	329.99	0.010
	5	500	10.15	4.93	6.74	4.20					
			10.12	4.99	6.77	4.17					
Avg		500	10.14	4.96	6.76	4.19	0.0165	408.57	198.08	357.34	0.008
	6	600	10.10	4.98	6.91	4.42					
			10.13	4.91	6.93	4.40					
Avg		600	10.12	4.95	6.92	4.41	0.0165	419.82	183.87	377.41	0.007
	7	700	10.14	4.97	7.05	4.58					
			10.11	4.91	7.07	4.60					
Avg		700	10.13	4.94	7.06	4.59	0.0165	428.74	173.76	391.96	0.007
	8	800	10.09	4.91	7.17	4.74					
			10.12	4.98	7.15	4.72					
Avg		800	10.11	4.95	7.16	4.73	0.0165	434.38	161.90	403.08	0.006

Part 2(h)

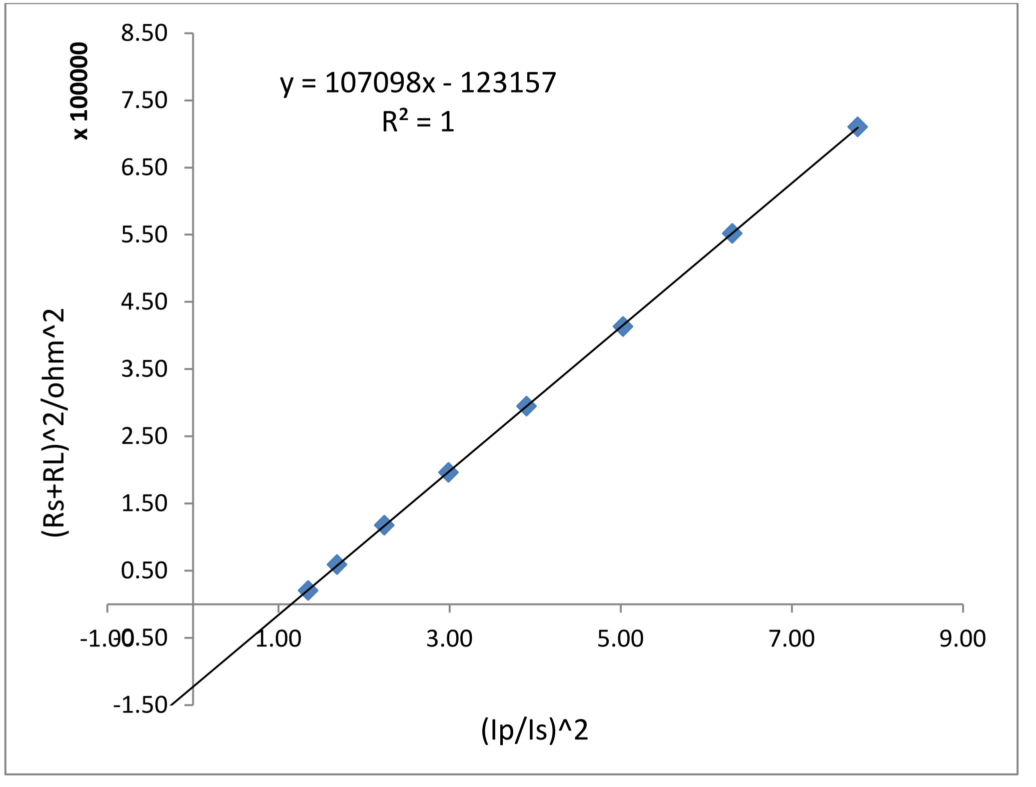
$$(R_S + R_L)^2 = (\omega M)^2 \left(\frac{I_P}{I_S}\right)^2 - X_S^2$$

Slope		M
1.07E+05		52.08 mH
Intercept		X_2
1.23E+05		350.94 Ω

Part 2(i)

$(Ip/Is)^2$	$(Rs+R_L)^2$
1.35	20438.25
1.69	59030.73
2.24	117623.21
2.99	196215.69
3.90	294808.17
5.03	413400.65
6.31	551993.13
7.77	710585.61

Part 2(j):



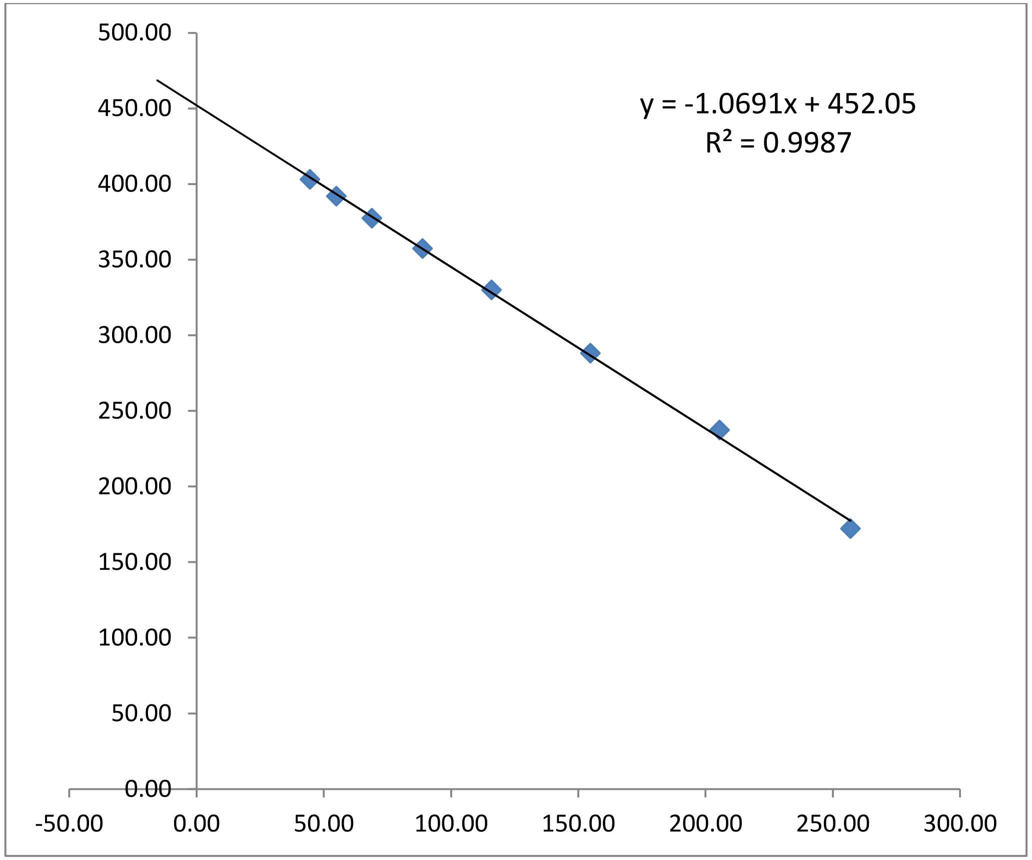
Part 3(k) and 3(l):

$$R_R = \left(\frac{I_S}{I_P}\right)^2 (R_S + R_L)$$

$$X_P = \left(\frac{I_S}{I_P}\right)^2 X_S$$

Sr.	R_R	X_R
1	106.00	256.95
2	144.08	205.52
3	153.17	154.78
4	148.17	115.92
5	139.16	88.82
6	127.84	68.91
7	117.81	54.95
8	108.46	44.59

Part 3(m)



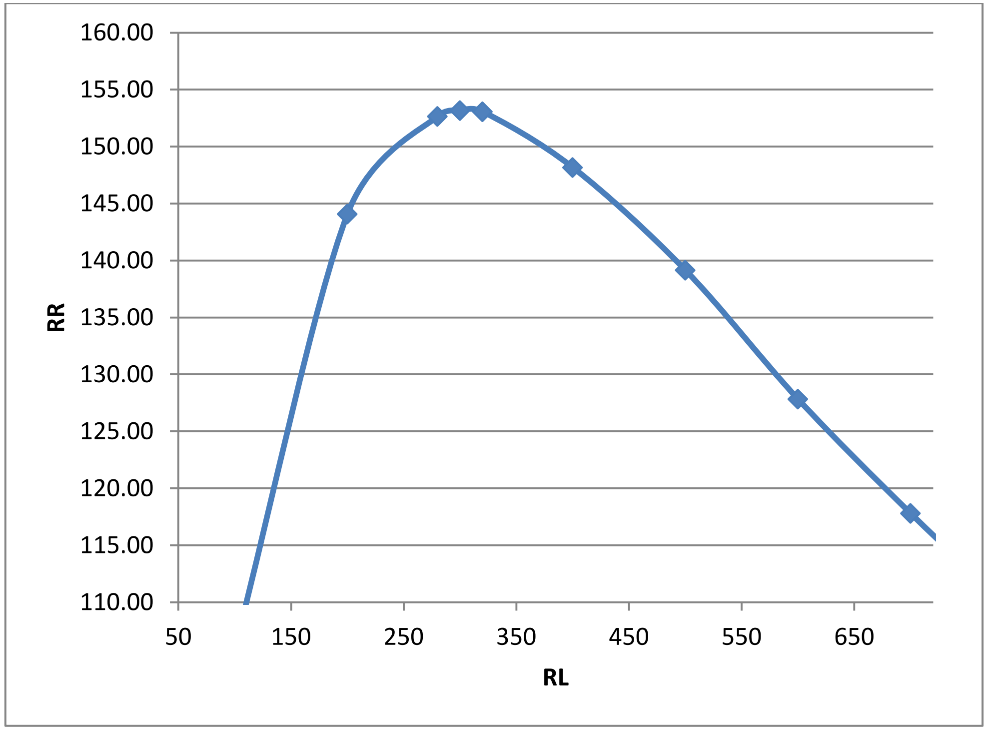
Inference:

$$X_P - X_R = X_{PE}$$

Part 3(n):

Sr.	R_L	V_A	$V_{R'}$	V	Vo	Ip	Is	$(Ip/Is)^2$	R_R
1	280	10.06	5.11	6.05	3.29				
		10.03	5.16	6.03	3.30				
	280	10.05	5.14	6.04	3.30	0.0171	0.01	2.12	152.65
2	320	10.02	5.09	6.19	3.51				
		10.05	5.03	6.21	3.50				
	320	10.04	5.06	6.20	3.51	0.0169	0.01	2.37	153.07

R_L	RR
100	106.00
200	144.08
280	152.65
300	153.17
320	153.07
400	148.17
500	139.16
600	127.84
700	117.81
800	108.45771



Part 4(o):

$$R_R = R^* - R_P$$

$$X_R = X_P - X^*$$

$$\frac{X_{core}}{R_{core}} = \frac{X_R}{R_R}$$

$$\frac{L_{core}}{R_{core}} = \frac{1}{2\pi f}\frac{\left(X_P - X^*\right)}{\left(R^* - R_P\right)}$$

Blue Coil	Green Coil
Lc/Rc	Lc/Rc
4.85E-04	4.50E-04

Part 4(p)

$$\Delta P = I_p^2\left(R_{PE} - R_P\right) - I_S^2\left(R_S + R_L\right)$$

		R_L	V_A	$V_{R'}$	V	Vo	I_1	I_2	R_{PE}
		1000	9.00	5.32	5.08	3.45			
			9.01	5.27	5.10	3.43			
Avg		1000	9.01	5.30	5.09	3.44	0.0177	0.003	145.23
			Δ P =	1.65E-02 W					