(Full Mark = 12 points)

Part A: Alignment of the setup

<table>
<tr><th>Tasks</th><th></th><th>Mark</th></tr>
<tr>
<td>A1</td>
<td>
Measure the height of the “zero-order spot” h, from the origin of the x-axis.
Determine the incident angle θ of the laser beam from D and h. Write down the values of h in cm and θ in degrees to three significant figures in the corresponding table in the answer sheet.

<u>Solution:</u>

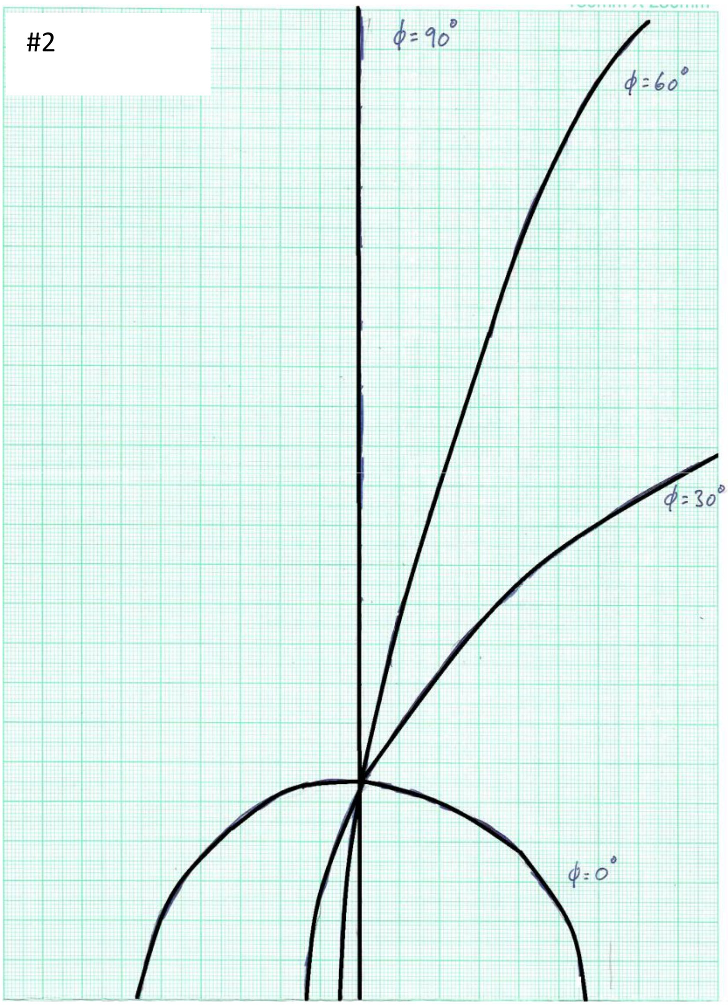
The height of the “zero-order spot” is measured to be h = 5.50 cm. Using trigonometry,
$$h = D\tan\theta$$
$$\theta = \tan^{-1}\left(\frac{h}{D}\right)$$
As h = 5.5 cm and D = 15 cm, one can then fill in the table as follows:
<table>
<tr><td>h</td><td>5.50 cm</td></tr>
<tr><td>θ</td><td>20.1°</td></tr>
</table>
</td>
<td>
TOTAL = 0.6 points

A1.1
0.2 points for measuring the correct height (between 5.40 cm to 5.60 cm)

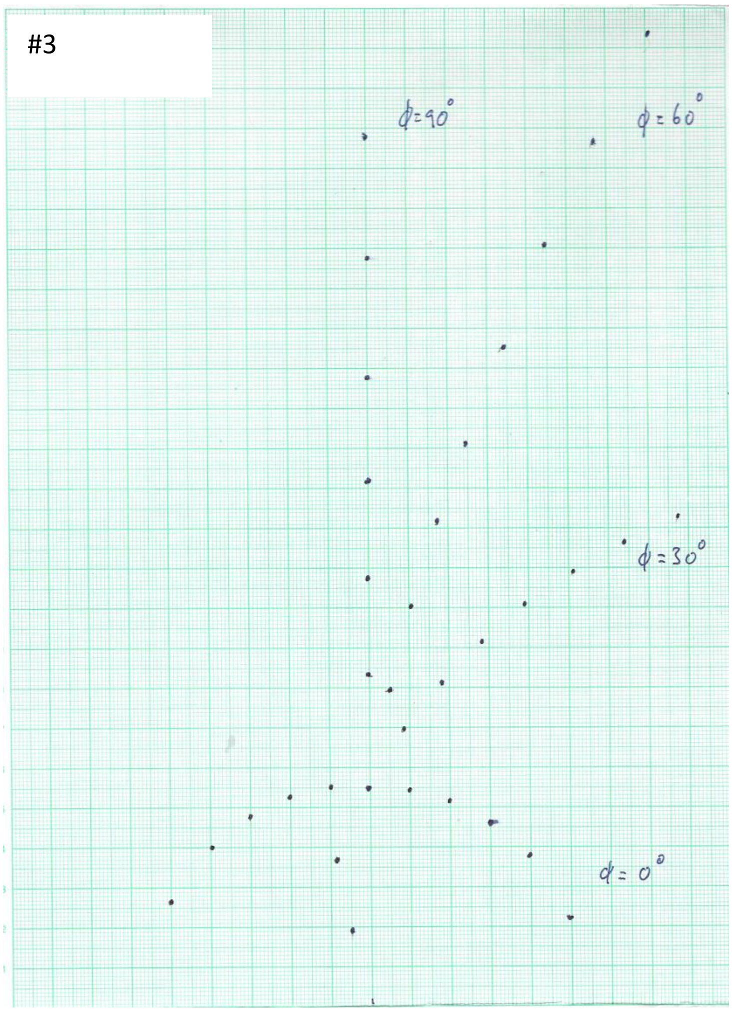
A1.2
0.2 points for using the correct equation for determining θ

A1.3
0.2 points for the correct value of θ
(0.2 points: θ is between 19.7° to 20.5°)
(Otherwise, 0 point)
</td>
</tr>
</table>

Part B: Diffraction patterns from Sample 2

Tasks		Mark
B1	**Record the diffraction patterns onto the graph paper for $\phi = 0^\circ, 30^\circ, 60^\circ$ and 90° of the rotary disk and write down their corresponding angle of rotation ϕ next to each pattern. Write down "# 2" at the top of this graph paper.** **Solution:** 	**TOTAL = 0.8 points** **B1.1** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 0^\circ$ **B1.2** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 30^\circ$ **B1.3** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 60^\circ$ **B1.4** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 90^\circ$

Part C: Diffraction patterns from Sample 3

Tasks		Mark
C1	**Mark the centers of the diffraction spots from Sample 3 for $\phi = 0^\circ, 30^\circ, 60^\circ$ and 90° onto the graph paper and write down their corresponding angle of rotation ϕ next to each pattern. Write down "# 3" at the top of this graph paper.** **Solution:** 	**TOTAL = 0.8 points** **C1.1** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 0^\circ$ **C1.2** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 30^\circ$ **C1.3** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 60^\circ$ **C1.4** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 90^\circ$

Part D: Theory behind the reflected diffraction patterns from Sample 3

<table>
<tr><th>Tasks</th><th></th><th>Mark</th></tr>
<tr><td>D1</td><td>
Equation (3) can be rearranged to obtain a quadratic equation for the grating constant a of Sample 3, as

$$Aa^2 + Ba + C = 0 \qquad \textbf{(4)}$$

Derive the expressions for A, B and C. Enter your results in the corresponding table in the answer sheet.

<u>Solution:</u>

The solution of D1 can be rearranged into the following form

$$[y^2\cos^2\theta + D^2\cos^2\theta - D^2]a^2 + [-2m\lambda\cos\theta\,(y^2 + D^2)]a + [m^2\lambda^2(y^2 + D^2)] = 0$$

Thus, we have

<table>
<tr><td><i>A</i></td><td>$y^2\cos^2\theta + D^2\cos^2\theta - D^2$</td></tr>
<tr><td><i>B</i></td><td>$-2m\lambda\cos\theta\,(y^2 + D^2)$</td></tr>
<tr><td>C</td><td>$m^2\lambda^2(y^2 + D^2)$</td></tr>
</table>

Other equivalent forms are also acceptable, such as:

<table>
<tr><td><i>A</i></td><td>$\cos^2\theta - \frac{D^2}{y^2 + D^2}$</td></tr>
<tr><td><i>B</i></td><td>$-2m\lambda\cos\theta$</td></tr>
<tr><td>C</td><td>$m^2\lambda^2$</td></tr>
</table>

<table>
<tr><td><i>A</i></td><td>$\frac{D^2}{y^2 + D^2} - \cos^2\theta$</td></tr>
<tr><td><i>B</i></td><td>$2m\lambda\cos\theta$</td></tr>
<tr><td>C</td><td>$-m^2\lambda^2$</td></tr>
</table>
</td><td>
TOTAL = 0.9 points

D1.1
0.3 points for getting the solution of Task (D1) by correct rearrangement

D1.2
0.2 points for the correct form of A

D1.3
0.2 points for the correct form of B

D1.4
0.2 points for the correct form of C

If <i>A</i>, <i>B</i> and <i>C</i> are all correct, then 0.9 points are still given.
</td></tr>
</table>

D2

By solving this quadratic equation and using the measured y values of the diffraction spots for Sample 3 at $\phi = 90^0$ (See Task C1), together with the known values of D, θ and λ, determine the grating constant a of Sample 3 in meters to three significant figures for each diffraction order from the 1st order (m = 1) up to the 6th order (m = 6) [*Hints: These orders correspond to the six spots above the zero-order spot*]. Enter your results in the corresponding table in the answer sheet.

<u>Solution:</u>

For each order m, we can construct the following table for the coefficients of A, B and C by using the measured value of y, the known values of D, $\cos\theta$ and λ (i.e. $D = 15\ \mathrm{cm}$, $\cos\theta = \cos 20.1^\circ = 0.939$ and $\lambda = 650\ \mathrm{nm}$):

Order m	**Measured value of y (meters)**	A	B	C
1	0.0835	3.481×10^{-3}	-3.567×10^{-8}	1.245×10^{-14}
2	0.1075	7.522×10^{-3}	-8.314×10^{-8}	5.756×10^{-14}
3	0.1315	1.258×10^{-2}	-1.457×10^{-7}	1.513×10^{-13}
4	0.1580	1.934×10^{-2}	-2.317×10^{-7}	3.209×10^{-13}
5	0.1875	2.833×10^{-2}	-3.519×10^{-7}	6.090×10^{-13}
6	0.2180	3.923×10^{-2}	-5.128×10^{-7}	1.065×10^{-12}

The standard solution for the quadratic equation of a is

$$a = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

which leads to two possible solutions, which are

$$a_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$a_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

Using the values of A, B and C from the table above, the values

TOTAL = 1.8 points

D2.1
0.4 points for calculating the values of A, B and C for all six orders

D2.2
0.4 points for presenting the correct solution of the quadratic equation

D2.3
0.4 points for providing a_1 and a_2 for all six orders

D2.4
0.4 points for pointing out that only a_1 is the correct solution with explanation

D2.5
0.2 points for the correct values of a for all six orders
(0.2 points: a is within the uncertainty of $\pm$*10%)*
(0.1 points: a is within the uncertainty of $\pm$*20%)*
(Otherwise, 0 point)

of a_1 and a_2 can be calculated for each order m as shown in the following table:

Order m	a_1 **(meters)**	a_2 **(meters)**
1	9.945×10^{-6}	3.578×10^{-7}
2	1.029×10^{-5}	7.415×10^{-7}
3	1.042×10^{-5}	1.154×10^{-6}
4	1.038×10^{-5}	1.565×10^{-6}
5	1.033×10^{-5}	2.072×10^{-6}
6	1.048×10^{-5}	2.587×10^{-6}

As shown above, all the values of a_1 are similar to each other. However, this is not the case for the values of a_2 . Thus a_2 is not a valid solution for the grating constant and should be discarded.

Order m	**Grating constant** a **(meters)**
1	9.95×10^{-6}
2	1.03×10^{-5}
3	1.04×10^{-5}
4	1.04×10^{-5}
5	1.03×10^{-5}
6	1.05×10^{-5}

Alternatively, one can also rearrange Equation (3) to the following form:

$$a = \frac{m\lambda}{\left(\cos\theta - \sqrt{\dfrac{D^2}{y^2 + D^2}}\right)}$$

and then solve for the grating constant a accordingly.

<table>
<tr><td>D3</td><td>

Calculate the mean for the grating constant a in meters to three significant figures and the standard error of the mean. Enter your results in the corresponding table in the answer sheet.

Solution:

As we have six values of a_1 (i.e. $n = 6$), the mean of the grating constant $\bar{a}$ can be calculated by

$$\bar{a} = \frac{\sum a_1}{n}$$

For the standard error of the mean for the grating constant, we can use

$$\sigma_{\bar{a}} = \sqrt{\frac{\sum_m (a_{1,m} - \bar{a})^2}{n-1}} / \sqrt{n}$$

Mean of grating constant $\bar{a}$	1.03×10^{-5} m
Standard error of the mean $\sigma_{\bar{a}}$	8×10^{-8} m

</td><td>

TOTAL = 0.8 points

D3.1
0.2 points for the correct formula in determining the mean

D3.2
0.2 points for the correct formula in determining the standard error of the mean

D3.3
0.2 points for the correct value of the mean
(0.2 points: $\bar{a}$ is within the uncertainty of $\pm 10\%$,
0.1 points: $\bar{a}$ is within the uncertainty of $\pm 20\%$,
otherwise, 0 point)

D3.4
0.2 points for the correct value of the standard error of the mean
(0.2 points: $\sigma_{\bar{a}}$ is within the uncertainty of $\pm 10\%$,
0.1 points: $\sigma_{\bar{a}}$ is within the uncertainty of $\pm 20\%$,
otherwise, 0 point)

</td></tr>
</table>

Part E: Determination of the unknown angle ϕ^* for Sample 4

Tasks		Mark
E1	**Along the continuous diffracted curve of Sample 4 projected on the graph paper, measure the y-coordinates in cm for ten points starting from x = - 1.0 cm to 3.5 cm with a step of 0.5 cm. Enter your results in the corresponding table in the answer sheet.**	**TOTAL = 0.6 points** **E1** 0.6 points for filling in the correct values of y-coordinate in the table.

Detailed marking allocation of Task E1 is as follows:

x /cm	*y* /cm	*y* /cm	*y* /cm
	±0.4cm ***0.6 pts***	±0.8cm ***0.4 pts***	±1.2cm ***0.2 pts***
-1.0	3.1 to 3.9	2.7 to 4.3	2.3 to 4.7
-0.5	4.1 to 4.9	3.7 to 5.3	3.3 to 5.7
0.0	4.8 to 5.6	4.4 to 6.0	4.0 to 6.4
0.5	5.6 to 6.4	5.2 to 6.8	4.8 to 7.2
1.0	6.1 to 6.9	5.7 to 7.3	5.3 to 7.7
1.5	6.6 to 7.4	6.2 to 7.8	5.8 to 8.2
2.0	6.9 to 7.7	6.5 to 8.1	6.1 to 8.5
2.5	7.3 to 8.1	6.9 to 8.5	6.5 to 8.9
3.0	7.6 to 8.4	7.2 to 8.8	6.8 to 9.2
3.5	7.9 to 8.7	7.5 to 9.1	7.1 to 9.5

Solution:

***x* co-ordinate (cm)**	***y* co-ordinate (cm)**
-1.0	3.5
-0.5	4.5
0.0	5.2
0.5	6.0
1.0	6.5
1.5	7.0
2.0	7.3
2.5	7.7
3.0	8.0
3.5	8.3

E2

Based on Eq. (1) given in Task (D), construct a linear equation in the form of

$$\boldsymbol{M(y,x,D,\theta) = I(D) + S(\phi^*)x\,.} \qquad \textbf{(4)}$$

Determine the functional forms for $\boldsymbol{M(y,x,D,\theta)}$, $\boldsymbol{I(D)}$ and $\boldsymbol{S(\phi^*)}$. Plot $\boldsymbol{M}$ against $\boldsymbol{x}$, using the data recorded in the table of Task (E1). Determine the unknown angle $\boldsymbol{\phi^*}$ in degrees from this graph. Write down all the functional forms and the value of $\boldsymbol{\phi^*}$ in the corresponding table in the answer sheet.

<u>Solution:</u>

Eq. (1) can be rearranged to get

$$\frac{(D\cos\phi^* + x\sin\phi^*)^2}{(\cos\theta\cos\phi^*)^2} = y^2 + x^2 + D^2$$

Take the square root for both sides and then multiply the right hand side by $\cos\theta$ to get

$$\cos\theta\sqrt{y^2+x^2+D^2} = D + (\tan\phi^*)\,x$$

Thus, the above equation can be rewritten as

$$M(y,x,D,\theta) = I(D) + S(\phi^*)x$$

$\boldsymbol{M(y,x,D,\theta)}$	$\cos\theta\sqrt{y^2+x^2+D^2}$
$\boldsymbol{I}$	D
$\boldsymbol{S}$	$\tan\phi^*$

If one plots $\cos\theta\sqrt{y^2+x^2+D^2}$ versus x, the slope of the resulting straight line will be $\tan\phi^*$. From the slope, one can find the unknown angle ϕ^*. Using the data from the table below,

x (cm)	**y (cm)**	**D (cm)**	**$\cos\theta$**	**$M = \cos\theta\sqrt{y^2+x^2+D^2}$**
-1.0	3.5	15	0.939	14.494
-0.5	4.5	15	0.939	14.713
0.0	5.2	15	0.939	14.907
0.5	6.0	15	0.939	15.177
1.0	6.5	15	0.939	15.379
1.5	7.0	15	0.939	15.607
2.0	7.3	15	0.939	15.777
2.5	7.7	15	0.939	16.005
3.0	8.0	15	0.939	16.210
3.5	8.3	15	0.939	16.430

TOTAL = 1.6 points

E2.1
0.3 points for the correct form of $M(y,x,D,\theta)$

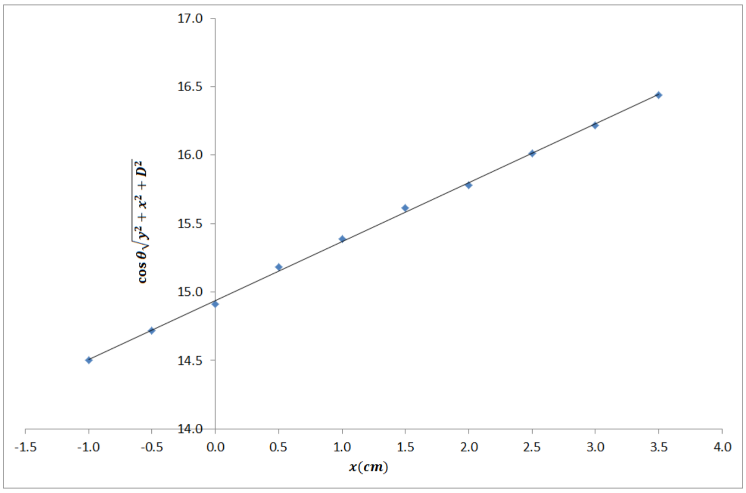
E2.2
0.3 points for the correct form of I

E2.3
0.3 points for the correct form of S

E2.4
0.4 points for plotting the linear relationship between M and x

E2.5
0.3 points for the correct value of ϕ^*
(0.3 points within the uncertainty of $\pm 5^\circ$,
0.2 points within the uncertainty of $\pm 8^\circ$,
otherwise, 0 point)

We can then make a plot of M vs x as shown below:



From the plot, we can estimate the slope to get $\phi^* = 23.2^\circ$. (Remark: one can also find out the y-intercept to be $D = 14.9$ cm, which is in good agreement with the value assigned in this experiment.)

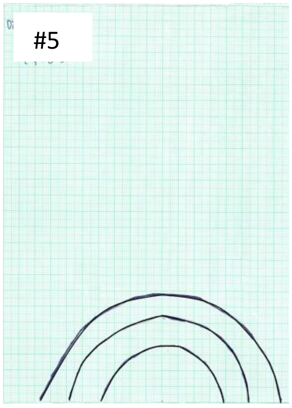
Therefore,

$\boldsymbol{\phi}^*$	23.2°

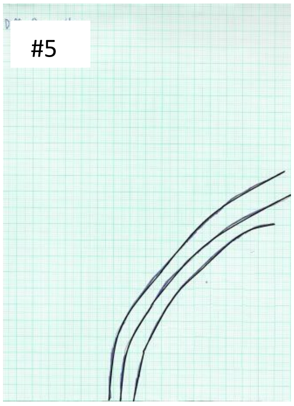
Part F: Diffraction patterns from Sample 5

Tasks		Mark
F1	**Record the diffraction patterns you observed for $\phi = 0^\circ, 30^\circ, 60^\circ$ and 90° on separate graph papers for each value of ϕ. At the top of each graph paper, put down '#5' and the corresponding ϕ value. It is expected that you could observe more than 10 diffraction orders. However, you are required to record only three relatively brighter orders on each graph paper.**	**TOTAL = 0.8 points** **F1.1** 0.2 points for the correct sketch of the diffraction pattern for $\phi = 0^\circ$ **F1.2** 0.2 points for the correct sketch of the diffraction pattern $\phi = 30^\circ$ **F1.3** 0.2 points for the correct sketch of $\phi = 60^\circ$ **F1.4** 0.2 points for the correct sketch of $\phi = 90^\circ$

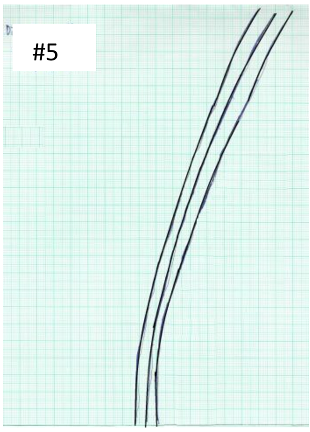
Solution:



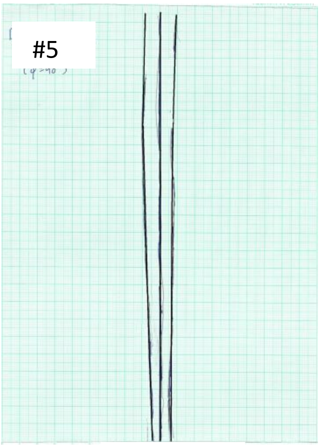
$\phi = 0^{\circ}$



$\phi = 30^{\circ}$



$\phi = 60^{\circ}$



$\phi = 90^{\circ}$

F2

With this understanding, estimate the spacing *b* in meters of the uniformly spaced pre-made grooves of Sample 5 using the recorded diffraction pattern for $\phi = 0^\circ$ from Task (F1).
Enter the value of *b* in the answer sheet.

[Note that in estimating the value of b, you are only required to take the measured data of the first diffraction order and the estimated b should be rounded up to three significant figures.]

Solution:

The formation of the observed diffraction pattern from Sample 5 at $\phi = 0^\circ$ can be considered in the following way: the periodic pre-made grooves (perpendicular to the z-direction) form a set of discrete diffraction spots lying along the y-axis. However, each of these spots is extended to form an arc due to the background straight scratched grooves with non-uniform spacing and thus forming a multi-arc pattern as observed. With this understanding, one can take the peak of the brightest arc as the zero-order (which should be located at $D\tan 20.1^\circ = 5.5$ cm), and then measure the y_1 value of the peak of the 1st order diffraction pattern. This should satisfy Equation (3) as below:

$$y_1 = D\sqrt{\frac{b^2}{(b\cos\theta - \lambda)^2} - 1}$$

where b is the spacing of the pre-made grooves. From the recorded diffraction at $\phi = 0^\circ$ for Sample 5, the value of y_1 is measured to be 0.0695 m. Similar to what has been done in Task (D1), one can rearrange the equation for y_1 to form a quadratic equation as

$$Ab^2 + Bb + C = 0$$

Where

A	${y_1}^2\cos^2\theta + D^2\cos^2\theta - D^2 = 1.481 \times 10^{-3}$
B	$-2\lambda\cos\theta\,({y_1}^2 + D^2) = -3.320 \times 10^{-8}$
C	$\lambda^2({y_1}^2 + D^2) = 1.149 \times 10^{-14}$

Again, we take the only valid solution of b (See the solution of Task (D3) for detailed explanations) as

TOTAL = 1.6 points

F2.1
0.3 points for providing the correct value of y_1
(0.3 points: y_1 is within the uncertainty of $\pm 10\%$,
(0.2 points: y_1 is within the uncertainty of $\pm 20\%$
Otherwise: 0 marks)

F2.2
0.3 points for quoting Equation (3)

F2.3
0.2 points for quoting the expressions of A derived in Task (D2)

F2.4
0.2 points for quoting the expressions of B derived in Task (D2)

F2.5
0.2 points for quoting the expressions of C derived in Task (D2)

F2.6
0.4 points for providing the correct value of b
(0.4 points: b is within the uncertainty of $\pm 10\%$,
0.2 points: b is within the uncertainty of $\pm 20\%$,
otherwise, 0 point)

<table>
<tr>
<td></td>
<td>

$$b = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore b \approx 2.21 \times 10^{-5} \text{ m}$$

<table><tr><td><i>b</i></td><td>2.21×10^{-5} m</td></tr></table>

Alternatively, one can also rearrange Equation (3) to the following form:

$$b = \frac{m\lambda}{\left(\cos\theta - \sqrt{\frac{D^2}{y^2 + D^2}}\right)}$$

and then solve for the spacing b accordingly.

</td>
<td></td>
</tr>
</table>

Part G: Determination of the plane spacing for ZnSe

<table>
<tr>
<th>Tasks</th>
<th></th>
<th>Mark</th>
</tr>
<tr>
<td>G1</td>
<td>

For the ZnSe sample, based on Figure 16 and the experimental conditions given above, determine the lattice-plane spacing a^* of the periodic atomic lattice planes that are perpendicular to the nano-grooves with non-uniform spacing, in meters. Enter your result in the corresponding table in the answer sheet.

Solution:

Recalling Eq. (2) given in Task (D),

$$x = \frac{Dm\lambda \cos\phi}{a^* \cos\theta - m\lambda \sin\phi}$$

For the periodic atomic lattice planes, one has $\theta \approx 0^\circ$ and $\phi = 0^\circ$ and so Eq. (2) becomes

$$x = \frac{Dm\lambda}{a^*}$$

</td>
<td>

TOTAL = 1.7 points

G1.1
0.4 points for quoting Eq. (2) in Task (D)

G1.2
0.5 points for providing the expression for a^*(from $\Delta x = \frac{D\lambda}{a^*}$)

G1.3
0.2 points for measuring the correct value of Δx (i.e. 0.7 cm) from the diffraction streaks as shown in Figure 10.

G1.4
0.3 points for the correct value of λ (i.e. 0.1067×10^{-10} m)

G1.5

</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>
where a^* is the lattice plane spacing of the periodic atomic lattice planes that are perpendicular to nano-grooves with non-uniform spacing. Thus the average spacing of the RHEED streaks can be written as

$$\Delta x = \frac{D\lambda}{a^*}$$

which can be measured from the given RHEED pattern to be 0.7 cm. Given that $D = 0.26\,\mathrm{m}$ and λ can be calculated to be $0.1067 \times 10^{-10}\,\mathrm{m}$ using Equation (6). Thus, the required lattice plane spacing of ZnSe can be calculated as

$$a^* = \frac{0.26\,\mathrm{m}\ \times (0.1067 \times 10^{-10}\,\mathrm{m})}{0.007\,\mathrm{m}}$$

$$\therefore a^* = 3.96 \times 10^{-10}\,\mathrm{m}$$

<table>
<tr><td>a^*</td><td>$3.96 \times 10^{-10}\,\mathrm{m}$</td></tr>
</table>

Remark: For ZnSe, the actual plane spacing for the corresponding lattice plane is $a^* = 4 \times 10^{-10} m$.

Alternatively, you can also consider the corresponding lattice planes of ZnSe as a regular grating, and then using the standard formula for diffraction gratings to obtain the same value of a^*.
</td>
<td>
0.3 points for the correct value of a^*

(0.3 points: a^* is within the uncertainty of $\pm 0.31 \times 10^{-10}$ m,
otherwise, 0 point)

Note: The main source of error in determining a^* is the uncertainty in measuring the spacing of the streaks using a ruler (i.e. ± 0.005m).
</td>
</tr>
</table>

END

(Full Mark = 8)

<table>
<tr><th>Tasks</th><th>Description</th><th>Marks</th></tr>
<tr>
<td>1</td>
<td>
Table E2_1
<table>
<tr><th>θ
(Deg)</th><th>Voltage
(mV)</th><th>$\cos\theta$</th></tr>
<tr><td>60.0</td><td>432</td><td>0.500</td></tr>
<tr><td>59.5</td><td>426</td><td>0.508</td></tr>
<tr><td>59.0</td><td>427</td><td>0.515</td></tr>
<tr><td>58.5</td><td>431</td><td>0.522</td></tr>
<tr><td>58.0</td><td>435</td><td>0.530</td></tr>
<tr><td>57.5</td><td>438</td><td>0.537</td></tr>
<tr><td>57.0</td><td>440</td><td>0.545</td></tr>
<tr><td>56.5</td><td>440</td><td>0.552</td></tr>
<tr><td>56.0</td><td>440</td><td>0.559</td></tr>
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<tr><td>55.0</td><td>434</td><td>0.574</td></tr>
<tr><td>54.5</td><td>430</td><td>0.581</td></tr>
<tr><td>54.0</td><td>425</td><td>0.588</td></tr>
<tr><td>53.5</td><td>423</td><td>0.595</td></tr>
<tr><td>53.0</td><td>426</td><td>0.602</td></tr>
<tr><td>52.5</td><td>431</td><td>0.609</td></tr>
<tr><td>52.0</td><td>435</td><td>0.616</td></tr>
<tr><td>51.5</td><td>437</td><td>0.623</td></tr>
<tr><td>51.0</td><td>439</td><td>0.629</td></tr>
<tr><td>50.5</td><td>439</td><td>0.636</td></tr>
<tr><td>50.0</td><td>438</td><td>0.643</td></tr>
<tr><td>49.5</td><td>437</td><td>0.649</td></tr>
<tr><td>49.0</td><td>431</td><td>0.656</td></tr>
<tr><td>48.5</td><td>428</td><td>0.663</td></tr>
<tr><td>48.0</td><td>424</td><td>0.669</td></tr>
<tr><td>47.5</td><td>421</td><td>0.676</td></tr>
<tr><td>47.0</td><td>422</td><td>0.682</td></tr>
<tr><td>46.5</td><td>425</td><td>0.688</td></tr>
<tr><td>46.0</td><td>430</td><td>0.695</td></tr>
<tr><td>45.5</td><td>433</td><td>0.701</td></tr>
<tr><td>45.0</td><td>435</td><td>0.707</td></tr>
<tr><td>44.5</td><td>436</td><td>0.713</td></tr>
<tr><td>44.0</td><td>437</td><td>0.719</td></tr>
<tr><td>43.5</td><td>436</td><td>0.725</td></tr>
<tr><td>43.0</td><td>434</td><td>0.731</td></tr>
<tr><td>42.5</td><td>431</td><td>0.737</td></tr>
<tr><td>42.0</td><td>425</td><td>0.743</td></tr>
<tr><td>41.5</td><td>424</td><td>0.749</td></tr>
<tr><td>41.0</td><td>420</td><td>0.755</td></tr>
<tr><td>40.5</td><td>419</td><td>0.760</td></tr>
<tr><td>40.0</td><td>420</td><td>0.766</td></tr>
<tr><td>39.5</td><td>424</td><td>0.772</td></tr>
<tr><td>39.0</td><td>428</td><td>0.777</td></tr>
<tr><td>38.5</td><td>431</td><td>0.783</td></tr>
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<tr><td>37.5</td><td>434</td><td>0.793</td></tr>
<tr><td>37.0</td><td>434</td><td>0.799</td></tr>
<tr><td>36.5</td><td>434</td><td>0.804</td></tr>
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<tr><td>35.5</td><td>431</td><td>0.814</td></tr>
<tr><td>35.0</td><td>429</td><td>0.819</td></tr>
<tr><td>34.5</td><td>426</td><td>0.824</td></tr>
<tr><td>34.0</td><td>422</td><td>0.829</td></tr>
<tr><td>33.5</td><td>419</td><td>0.834</td></tr>
</table>
</td>
<td>
TOTAL = 1.0 points

0.6 points for two columns filled with values of θ and the voltage

0.2 points for including units

0.2 points for displaying the measurements with the correct number of significant figures

[-0.1 points for each unit missing

-0.1 points for displaying the incorrect number of significant figures for two or more variables

-0.1 points for angular step $\delta\theta > 2^{o}$]
</td>
</tr>
</table>

33.0	417	0.839
32.5	417	0.843
32.0	419	0.848
31.5	421	0.853
31.0	424	0.857
30.5	427	0.862
30.0	429	0.866
29.5	430	0.870
29.0	431	0.875
28.5	432	0.879
28.0	431	0.883
27.5	431	0.887
27.0	430	0.891
26.5	429	0.895
26.0	427	0.899
25.5	425	0.903
25.0	421	0.906
24.5	419	0.910
24.0	416	0.914
23.5	414	0.917
23.0	414	0.921
22.5	414	0.924
22.0	415	0.927
21.5	416	0.930
21.0	419	0.934
20.5	421	0.937
20.0	423	0.940
19.5	424	0.943
19.0	426	0.946
18.5	427	0.948
18.0	427	0.951
17.5	428	0.954
17.0	428	0.956
16.5	429	0.959
16.0	428	0.961
15.5	428	0.964
15.0	428	0.966
14.5	427	0.968
14.0	427	0.970
13.5	426	0.972
13.0	425	0.974
12.5	423	0.976
12.0	423	0.978
11.5	422	0.980
11.0	420	0.982
10.5	419	0.983
10.0	418	0.985
9.5	417	0.986
9.0	416	0.988
8.5	416	0.989
8.0	414	0.990
7.5	413	0.991
7.0	413	0.993
6.5	413	0.994
6.0	412	0.995
5.5	411	0.995
5.0	410	0.996
4.5	-	-
4.0	-	-

Reflection intensity against incident angle taken from θ ~ 4^o to 60^o in 0.5^o intervals on one (LHS) side of the angular scale. The independent variable $\cos\theta$ should be added to Table E2_1. (The Ti-coated etalon used is #15.)

2

Table E2_2

θ (Deg)	Voltage (mV)	cos θ
-60.0	426	0.500
-59.5	432	0.508
-59.0	437	0.515
-58.5	439	0.522
-58.0	441	0.530
-57.5	441	0.537
-57.0	440	0.545
-56.5	438	0.552
-56.0	434	0.559
-55.5	426	0.566
-55.0	421	0.574
-54.5	421	0.581
-54.0	427	0.588
-53.5	432	0.595
-53.0	435	0.602
-52.5	438	0.609
-52.0	439	0.616
-51.5	439	0.623
-51.0	437	0.629
-50.5	434	0.636
-50.0	429	0.643
-49.5	422	0.649
-49.0	418	0.656
-48.5	420	0.663
-48.0	425	0.669
-47.5	431	0.676
-47.0	434	0.682
-46.5	436	0.688
-46.0	436	0.695
-45.5	436	0.701
-45.0	436	0.707
-44.5	433	0.713
-44.0	431	0.719
-43.5	426	0.725
-43.0	421	0.731
-42.5	413	0.737
-42.0	417	0.743
-41.5	420	0.749
-41.0	423	0.755
-40.5	428	0.760
-40.0	431	0.766
-39.5	431	0.772
-39.0	433	0.777
-38.5	432	0.783
-38.0	433	0.788
-37.5	431	0.793
-37.0	422	0.799
-36.5	425	0.804
-36.0	417	0.809
-35.5	416	0.814
-35.0	414	0.819
-34.5	414	0.824
-34.0	419	0.829
-33.5	421	0.834
-33.0	425	0.839
-32.5	427	0.843
-32.0	428	0.848

TOTAL = 1.0 points

0.6 points for two columns filled with values of θ and the voltage

0.2 points for including units

0.2 points for displaying the measurements with the correct number of significant figures

[-0.1 points for each unit missing

-0.1 points for displaying the incorrect number of significant figures for two or more variables

-0.1 points for angular step $\theta > 2^o$]

-31.5	430	0.853
-31.0	431	0.857
-30.5	432	0.862
-30.0	431	0.866
-29.5	430	0.870
-29.0	428	0.875
-28.5	427	0.879
-28.0	424	0.883
-27.5	421	0.887
-27.0	418	0.891
-26.5	415	0.895
-26.0	413	0.899
-25.5	413	0.903
-25.0	413	0.906
-24.5	416	0.910
-24.0	418	0.914
-23.5	420	0.917
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-22.0	426	0.927
-21.5	427	0.930
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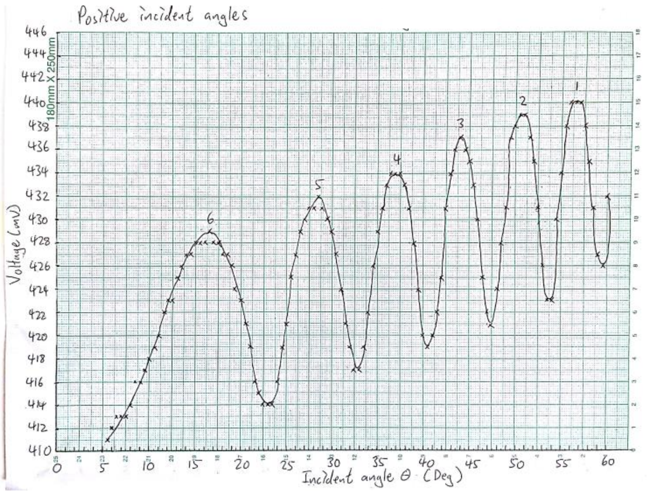
-20.0	429	0.940
-19.5	429	0.943
-19.0	428	0.946
-18.5	429	0.948
-18.0	428	0.951
-17.5	427	0.954
-17.0	426	0.956
-16.5	425	0.959
-16.0	423	0.961
-15.5	422	0.964
-15.0	419	0.966
-14.5	418	0.968
-14.0	416	0.970
-13.5	414	0.972
-13.0	414	0.974
-12.5	413	0.976
-12.0	412	0.978
-11.5	411	0.980
-11.0	411	0.982
-10.5	411	0.983
-10.0	411	0.985
-9.5	412	0.986
-9.0	412	0.988
-8.5	413	0.989
-8.0	414	0.990
-7.5	414	0.991
-7.0	415	0.993
-6.5	416	0.994
-6.0	416	0.995
-5.5	417	0.995
-5.0	418	0.996
-4.5	426	0.997
-4.0	425	0.998

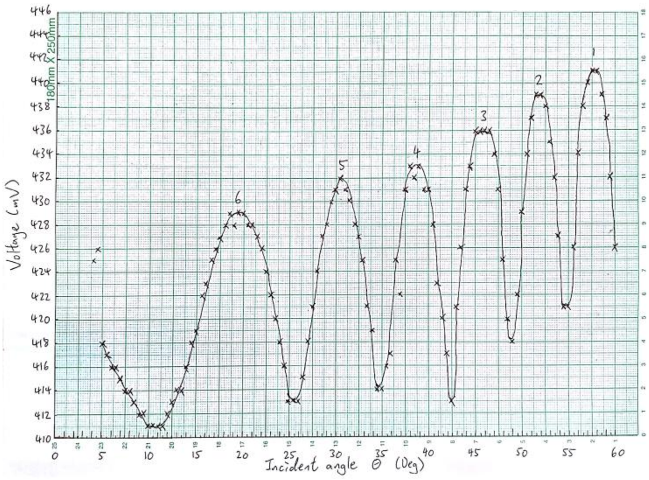
Reflection intensity against incident angle taken from θ ~ -4^{o} to -60^{o} in 0.5^{o} intervals on the (RHS) other side of the angular scale. The independent variable $\cos\theta$ should be added to Table E2_2. (The Ti-coated etalon used is #15.)

3

Graph E2_1



Graph E2_2



Graphs E2_1 and E2_2 show the relationship between the intensity and $|\theta|$ for the positive (LHS) and negative (RHS) incident angles respectively. The peak numbers are also labeled for all graphs.

TOTAL = 0.9 points

0.4 points for displaying the data points (for both graphs)

0.1 points for displaying the units (for both graphs)
0.1 points for displaying the axis label (for both graphs)

0.1 points for displaying the axis ticks label (for both graphs)

0.2 points for smooth curve (for both graphs)

[deduct half of the points if the above items are not shown in both graphs]

4	Refer to Graphs E2_1 and E2_2.	**TOTAL = 0.2 points** **0.2 points for labeling the peaks with appropriate peak numbers (for both graphs)** **[-0.1 points for wrong peak number order]**
5	From Equation (1), the peaks correspond to constructive interference where the total phase difference of the two beams is equal to multiples of 2π, i.e. $2kL\cos\theta_m + \phi_s = 2m\pi, \qquad (2)$ where m = 1, 2, etc… is the interference order and θ_m is the corresponding incident angle for peak reflection intensity. Thus a plot of the interference order m vs. cos θ_m will give a straight line with a slope related to the air-gap spacing L and an intercept related to the reflection phase ϕ_s. Since the interference order changes sequentially with the peak number, a plot of the peak number vs. cos θ will also give a straight line with the same slope as the plot for the interference order vs. cos θ. However, the y-intercept is now shifted along the y-axis with respect to the plot for the interference order vs. cos θ. Thus $X(\theta)$ = cos (θ) should be chosen as the independent variable such that the intensity peaks will be evenly spaced in a plot of reflection intensity vs. cos (θ). Furthermore, a plot of peak number vs. cos (θ) will give a straight line that can be used to obtain the air-gap spacing L of the Ti-coated etalon and also the reflection phase ϕ_s of the Ti.	**TOTAL = 0.3 points** **0.2 points for deriving the correct equation (i.e. Eq.(2))** **0.1 points for *m* equals to integer number**
6	Independent variable $X(\theta)$ = cos (θ) Refer to Tables E2_1 and E2_2	**TOTAL = 0.4 points** **0.2 points for correct independent $X(\theta)$** **0.2 points for working out the numbers of $X(\theta)$ in Tables E2_1 and E2_2**

7

Ideally, the locations of the peaks for (RHS) negative incident angles should be the same as for (LHS) positive incident angles for perfect opitcal alignment. Since there could be an offset in the locations of the peaks due to mis-alignment in the optics, the peak locations are better determined by averaging the negative and positive incident angles. After pairing, one would get the following table:

Table E2_3

Peak number (LHS)	θ_{LHS} **(Deg)**	**Peak number (RHS)**	θ_{RHS} **(Deg)**	$\|\theta\|_{average}$ **(Deg)**	$\cos\|\theta\|_{average}$	m
6	16.50	6	-19.25	17.875	0.952	11
5	28.50	5	-30.75	29.625	0.869	10
4	36.50	4	-38.50	37.500	0.793	9
3	44.00	3	-45.75	44.875	0.709	8
2	51.25	2	-51.75	51.500	0.623	7
1	56.50	1	-57.75	57.125	0.543	6

θ_{LHS} refers the peak location obtained from **Table E2_1.**
θ_{RHS} refers the peak location obtained from **Table E2_2.**
$|\theta|_{average}$ is the average of $|\theta|_{LHS}$ and $|\theta|_{RHS}$.

TOTAL = 0.6 points

0.2 points for identifying the peaks and the corresponding LHS/RHS incident angles

0.2 points for matching of peaks

0.2 points for calculating the average of the independent variable

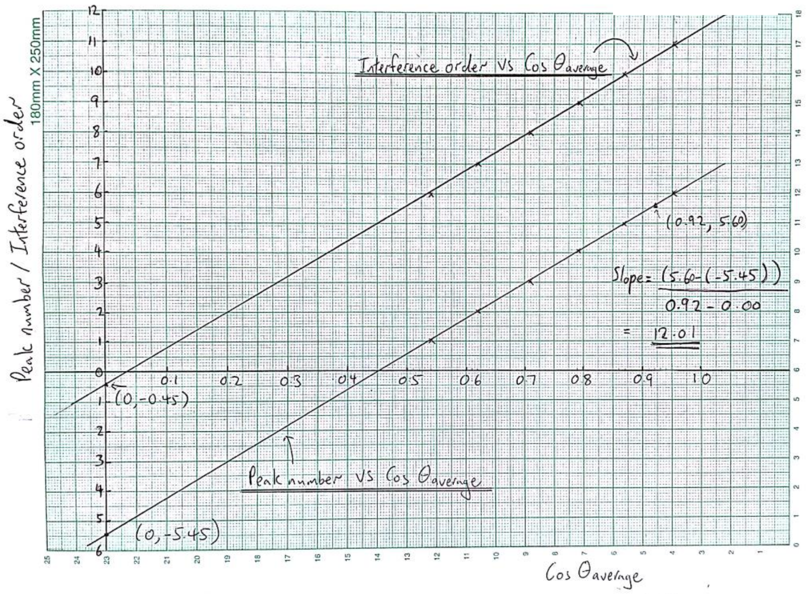
[-0.1 points for each unit missing

-0.1 points for displaying the incorrect number of significant figures except for peak and interference numbers

-0.1 points for mis-matching peak number]

8

Graph E2_3



Peak number vs. cos $\boldsymbol{\theta}_{\textbf{average}}$

The slope is 12.0

TOTAL = 0.6 points

0.3 points for displaying the data points

0.1 points for displaying the units

0.1 points for displaying the axis label

0.1 points for displaying the axis ticks label

[-0.2 points for plotting cos *θ* vs. peak number or interference order (i.e. *x-y* axis are reserved)]

The y-intercept is -5.45

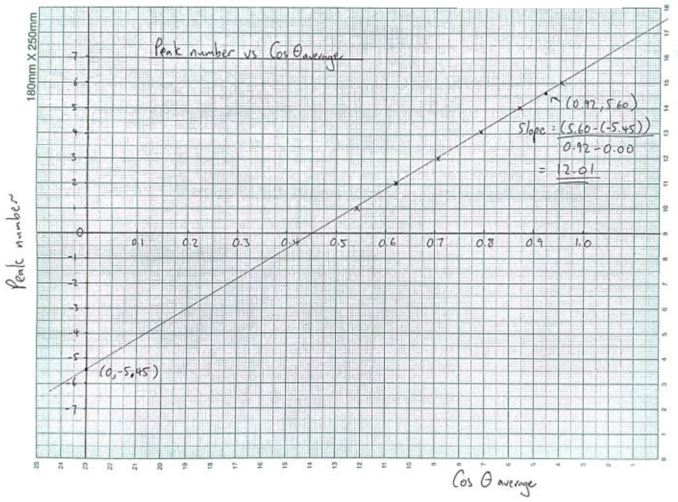
Interference order vs. cos θ_{average}

The slope is 12.0
The y-intercept is -0.45

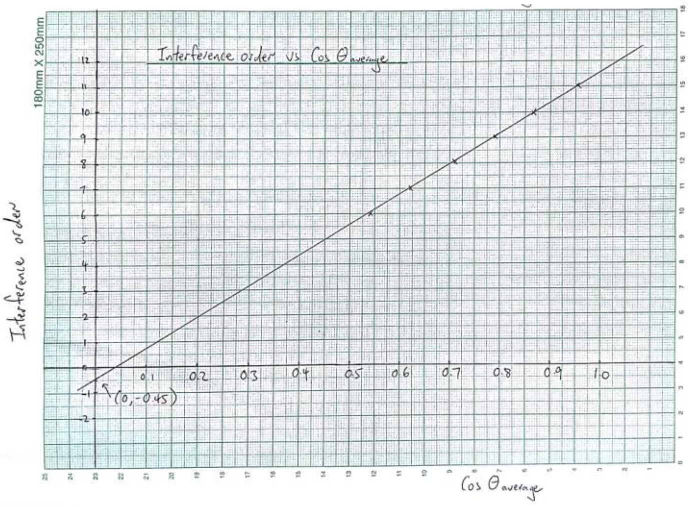
By plotting peak number against cos $|\theta|_{\text{average}}$ and drawing a line through the data points, one could get the slope and the y-intercept, as shown in Graph E2_3. The same principle applies to plotting the interference order m against cos $|\theta|_{\text{average}}$.
(Graphical solutions for the slope and intercept will be accepted.)

Note: Plotting these two graphs separately will also be acceptable as shown in Graph E2_3a and Graph E2_3b i.e.

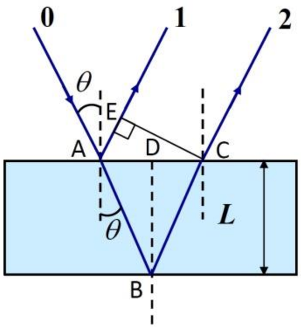
Graph E2_3a



Graph E2_3b



9	Refer to Graph E2_3 **Peak number vs. cos θ_{average}** The slope is 12.0 The y-intercept is -5.45	**TOTAL = 0.4 points** **0.2 points for fitting a straight line** **0.1 points for deriving the value of the slope** **0.1 points for deriving the value of the y-intercept**
10	Equation (2) in Tasks 5+6 can be rewritten in a simpler form for each constructive interference order m at incident angle θ_m as below: $m = \frac{2L\cos\theta_m}{\lambda} + \frac{\phi_s}{2\pi}\ .$ (3a) In Equation (3a), one can take the integer part of $\frac{2L\cos\theta_m}{\lambda}$ as the interference order, i.e. $m = \text{Trunc}\left(\frac{2L\cos\theta_m}{\lambda}\right)\ .$ (3b) Then the decimal part of $\frac{2L\cos\theta_m}{\lambda}$ is related to the reflection phase by $\phi_{\text{s,n}} \equiv \frac{\phi_s}{2\pi} = \text{Trunc}\left(\frac{2L\cos\theta_m}{\lambda}\right) - \frac{2L\cos\theta_m}{\lambda}.$ (3c) A simpler method to find the interference order m is to add to the peak number directly the absolute value of the integer part of the y-intercept obtained from the plot of peak number vs. cos θ as shown in Graph E2_3, i.e. m = peak number + |integer part of y-intercept in Graph E2_3. Then, the y-intercept of a plot of m vs. cos θ will give the normalized reflection phase $\phi_{\text{s,n}} = \phi_s/2\pi$ directly. $\phi_{\text{s,n}}$ is now defined within (-1, 0), corresponding to ϕ_s chosen with (-2π, 0). Refer to Table E2_3 for m.	**TOTAL = 1.2 points** **0.3 points for deriving the correct equation (i.e. Eq. (3a))** **0.3 points for getting the expression for m (i.e. Eq. (3b))** **0.2 points for getting the expression for normalized reflection phase (i.e. Eq. (3c))** **0.2 points for defining the range for the reflection phase** **0.2 points for interference order added to Table E2_3.**

11	Refer to Graph E2_3: From Graph E2_3, the slope for m vs, cos θ is 12.01. Since slope of the line is equal to $2L/\lambda$, one can then obtain the value of L as $$L = \lambda \times \text{ slope }/2 = 0.650 \times \frac{12.01}{2} = 3.903\ \mu\text{ m.}$$ The y-intercept is the normalized reflection phase $$\phi_{\text{s,n}} = \text{-0.450 or } \phi_{\text{s}} = \text{-2.828 rad.}$$	**Total = 1.4 points** **0.4 points for displaying the data points** **0.3 points for fitting a straight line** **0.4 points for deriving the value of the air-gap spacing** **0.3 points for deriving the value of the *y*-intercept** **[-0.1 points for $\phi_{\text{s,n}}$ outside (-0.9, -0.1)]**
	Appendix: Path difference calculation for an ideal air-gap etalon:  Path difference for beams 1 and 2 is equal to: $$\text{AB} + \text{BC} - \text{AE} = \frac{2L}{\cos\theta} - 2L\sin\theta\tan\theta = 2L\cos\theta. \qquad (4a)$$ This is the path difference used in Equation (1). It is also acceptable to calculate the path difference directly using Equation (1), but will only be given half of the points as writing down Eq. (4a). A mis-alignment of angle α between the laser beam and the angular scale	**N/A**

corresponds to a correction of α for the incident angle θ taken directly from the angular scale. Thus the incident angle is now $\theta + \alpha$. Hence the corrected path difference between beams 1 and 2 with a mis-alignment of angle α is:

$$\mathrm{AB} + \mathrm{BC} - \mathrm{AE} = 2L\cos(\theta + a). \qquad (4b)$$

Thus the correction of the path difference for a mis-alignment of angle α as compared to the ideal case of perfect alignment is:

$$\Delta = 2L\cos(\theta) - 2L\cos(\theta + a) \sim 2L\sin\theta\sin\alpha. \qquad (4c)$$

Hence, the error for the reflection phase due to mis-alignment is $2\pi\Delta/\lambda$, or Δ/λ when normalized by 2π.

Now one can use L = 5μm, θ =30°, and λ = 0.650 μm to get the error for the reflection phase due to a $\alpha = 1^{\circ}$ mis-alignment by using Equation (4c):

The value of Δ by using Equation (4c) is:

$$\Delta = 2 \times (5.00 \times 10^{-6}) \times \sin 30^{\circ} \times \sin 1^{\circ}$$

$$= 0.0873 \times 10^{-6}\ \mathrm{m}.$$

Now, the phase error is is $2\pi\Delta/\lambda$, or Δ/λ

$\sigma_{\phi_S} = 2\pi \times (0.0873 \times 10^{-6})/(0.650 \times 10^{-6}) = 0.843$ rad,

$\sigma_{\overline{\phi}_S} = 0.134$ (normalized by 2π).