

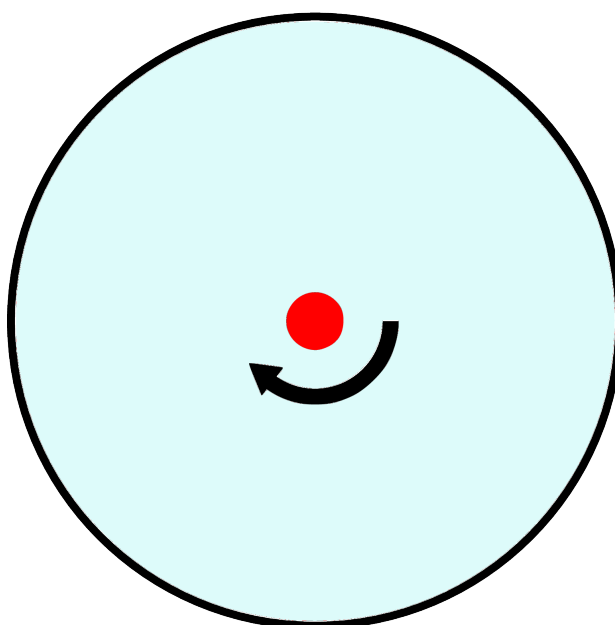


Vortices in Superfluid

Part A. Steady filament (0.75 points)

A.1 (0.25 pt)

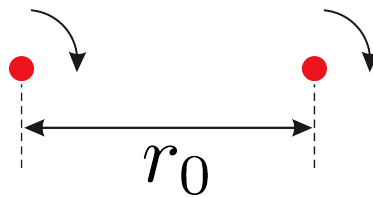
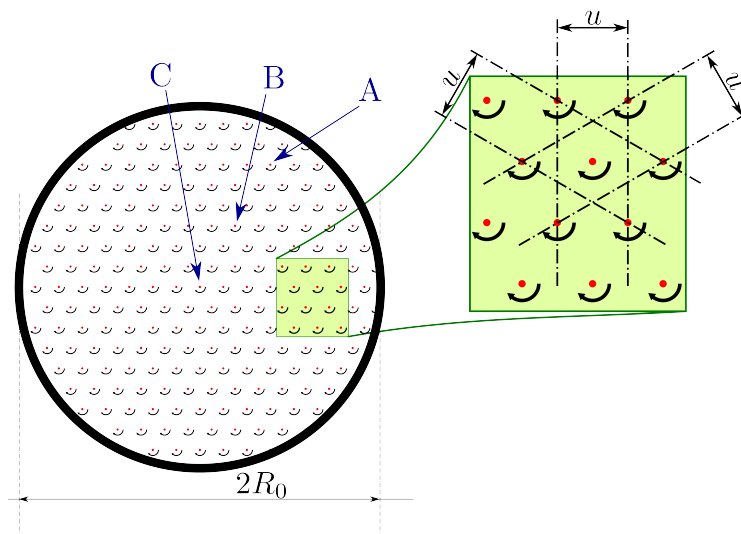
$v =$



A.2 (0.5 pt)

$z(r) =$


Part B. Vortex motion (1.4 points)
B.1 (0.25 pt)

 $v_0 =$

B.2 (0.15 pt)

B.3 (0.4 pt)

 $v(\vec{r}) =$
B.4 (0.35 pt)

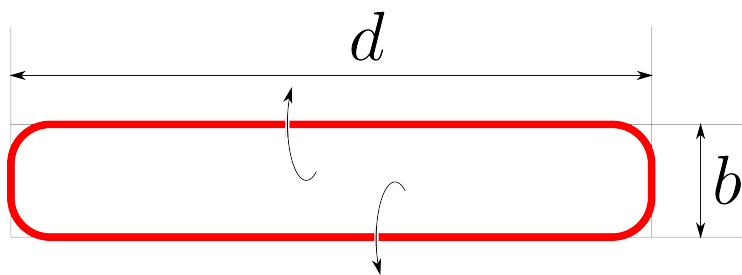
 $AB(t) =$


B.5 (0.25 pt)

$$z(\vec{r}) =$$

Part C. Momentum and Energy (1.75 points)
C.1 (0.3 pt)

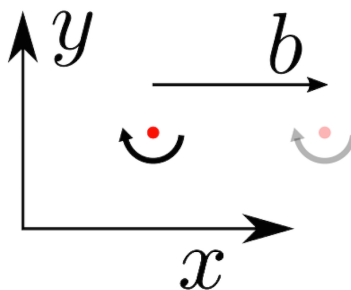
$$|\vec{P}| =$$


C.2 (0.7 pt)

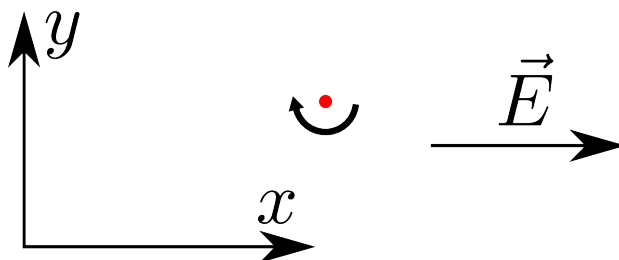
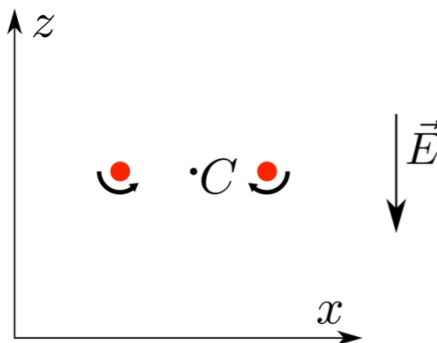
$$U =$$

C.3 (0.75 pt)

$$|\Delta\vec{P}| =$$




Part D. Trapped charges (2.85 points)
D.1 (0.5 pt)

 $v(t) =$

D.2 (0.6 pt)

 $R(t) =$
D.3 (1.5 pt)

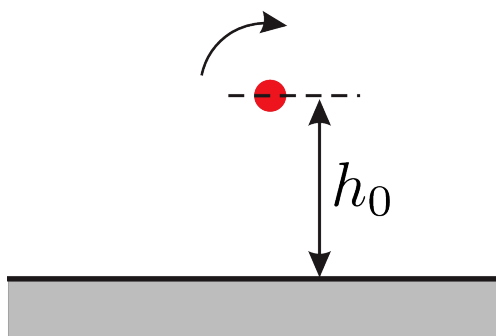
 $v(t) =$
D.4 (0.25 pt)

 $v(t) =$

Part E. Influence of the boundaries (3.25 points)

E.1 (0.5 pt)

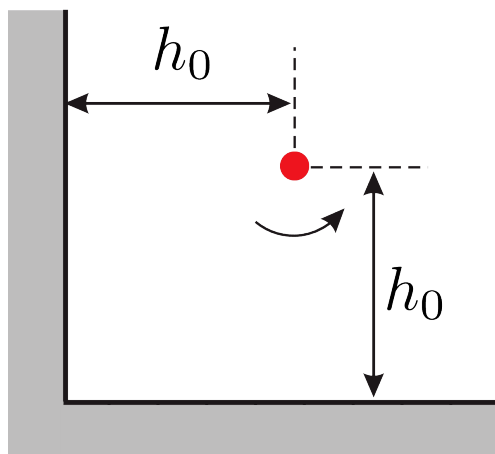
$v(t) =$



E.2 (0.75 pt)

$v_0 =$

E.3 (0.5 pt)



E.4 (1.5 pt)

$v_\infty =$

Evolution of Supermassive Black Holes Binary

Part A. Dynamic Friction

A.1 (0.75 pt)

$$k =$$

A.2 (0.25 pt)

$$\Delta p_x =$$

A.3 (0.4 pt)

$$F_{DF} =$$

A.4 (0.2 pt)

$$\log \Lambda =$$

Part B. Gravitational slingshot

B.1 (0.25 pt)

$$v_{bin} =$$

$$E =$$

B.2 (0.5 pt)

$$b =$$

B.3 (1.0 pt)

$$\Delta t =$$

**B.4** (0.25 pt)

$$\frac{dE}{dt} =$$

$$\left(\frac{da}{dt}\right)_{SS} =$$

B.5 (1.0 pt)*(formula)*

$$T_{SS} =$$

(value)

$$T_{SS} =$$

Part C. Emission of gravitational waves**C.1** (0.2 pt)

$$\left(\frac{da}{dt}\right)_{GW} =$$

C.2 (0.7 pt)

$$T_{GW} =$$

C.3 (0.1 pt)

$$a_H =$$

Part D. Full evolution.**D.1** (0.25 pt)

$$v =$$

D.2 (0.75 pt)

$$\left(\frac{da}{dt}\right)_{DF} =$$

**D.3** (0.3 pt)*(formula)* $a_1 =$ *(value)* $a_1 =$ **D.4** (0.75 pt) $T_1 =$ **D.5** (0.3 pt)*(formula)* $a_2 =$ *(value)* $a_2 =$ **D.6** (1.75 pt) $T_2 =$ $T_3 =$ **D.7** (0.3 pt) $T_{ev} =$



Space Debris

Part A. Rotation (3.8 points)

A.1 (0.2 pt)

$$\omega_x =$$

$$\omega_y =$$

A.2 (0.4 pt)

$$E_x =$$

$$E_y =$$

$$E(L, \theta) =$$

A.3 (1.2 pt)

$$\psi =$$

A.4 (2.0 pt)

$$\Omega(t) =$$

$$\gamma_s(t) =$$

$$\omega_s(t) =$$

Part B. Transient process (1.2 points).

B.1 (0.6 pt)

$$\theta_2 =$$

B.2 (0.6 pt)

$$\omega_2 =$$

**Part C. Magnetic Field (5.0 points).****C.1** (1.0 pt)

$$\mu_x =$$

$$\mu_y =$$

$$\mu_z =$$

C.2 (0.3 pt)

$$M_x =$$

$$M_y =$$

$$M_z =$$

C.3 (0.4 pt)

$$B_{EX} =$$

$$B_{EY} =$$

$$B_{EZ} =$$

C.4 (1.3 pt)

$$M_x(u) =$$

$$M_y(u) =$$

$$M_z(u) =$$

C.5 (1.0 pt)

$$\omega(t) =$$

C.6 (1.0 pt)

$$T/T_s =$$