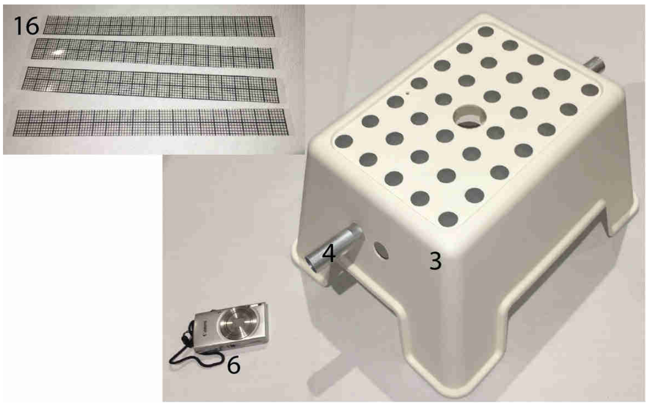
Static response of a magnetically active fluid (10 points)

Introduction

Ferrofluids are suspensions of nanoparticles of magnetite (Fe_3O_4) in a carrier medium. They exhibit a range of interesting properties, and in particular a strong response to an applied magnetic field - they are sometimes described as superparamagnets. In this experiment you will be investigating empirically some of the properties of a ferrofluid using both static and dynamic testing methods, and using a range of experimental measurement and estimation techniques. The experiment is set up in two broad halves but it is suggested to work through the problems in order.

Equipment List



1. Small glass bottle with ferrofluid under a clear medium. **You are NOT permitted to open the bottle for any reason!**
2. Glass dish with sealed lid containing ferrofluid. **You are NOT permitted to open the lid of the dish for any reason!**
3. A combination light shroud and stand
4. Tube with adjustable magnet carrier (initially inserted through the stand)
5. Adjustable wooden base with nylon bolts for level adjustment
6. Camera with inserted memory card
7. 2 × N52 magnet, 14.2 mm × 3.2 mm
8. 1 × N42 magnet, 20.0 mm × 5.0 mm (not shown in figure)
9. 500 mm threaded rod for use as a lamp pole
10. Washer and wing nut for attaching pole to stand
11. Battery-powered lamp with threaded mounting hole
12. Aluminium foil for making light guides and blockers
13. Blutack for attaching various components as needed
14. Green wedge
15. Transparent 30 cm ruler
16. 4 × transparent grid strips
17. 2 × wooden spacers
18. Paper tape measure

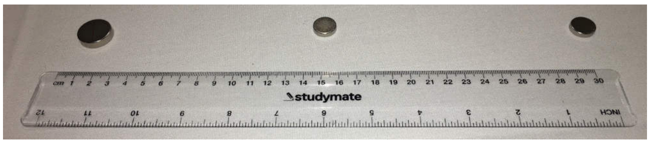
Safety and Other Important Notes

In this experiment you will be using high strength magnets. If you allow the magnets to attract each other they may pinch your fingers, or collide and shatter. Be careful to control the magnets at all times and not leave them near to each other unattended. **Broken magnets will not be replaced.**

This experiment has two sealed chambers of fluid. You may not open either the small glass bottle or the glass dish with sealed lid at any time during the experiment. **If the sides of the dish become coated with ferrofluid, they will be very hard to see through so be careful not to tip the dish unnecessarily.**

Your lamp is battery operated. If you deem it necessary, you may ask for **one** additional set of batteries for your lamp when you need them during the experiment.

If you hold a magnet close against the ferrofluid for more than around ten seconds, it will cause the fluid to behave differently due to residual magnetisation and interactions with the surrounding fluid. Although your experiment can still be performed, it may be more difficult to see the right effects.



When you store your magnets, ensure that they are far enough apart so that they won't snap together. They must be placed at least as far apart as is shown in the diagram above.

Part A: Static Testing (1.6 points)

In this part of the experiment, you will be investigating the ferrofluid properties through measurements of its static response to a magnetic field.

Magnetic interaction: force on a ferrofluid

The small glass bottle contains a volume of ferrofluid surrounded by an unknown fluid with which the ferrofluid is immiscible. The ferrofluid has density 1.21×10^3 kg m^{-3}, and magnetic susceptibility $\chi = 2.64$.

The ferrofluid responds to the presence of a magnetic field **B** and has an induced dipole moment per volume of magnitude $m = \frac{\chi B}{\mu_0}$.

The field on the axis of a cylindrical magnet is approximately

$$B_z = \frac{B_r}{2}\left(\frac{z+l}{\sqrt{(z+l)^2+a^2}} - \frac{z}{\sqrt{z^2+a^2}}\right) \quad , \tag{1}$$

where z is the distance from the surface of the magnet, l is its thickness, a its radius and B_r the remanent field strength which is a property of the magnetic material from which the magnet is made. For the large magnet (made of N42) $B_r = 1.3$T and for the small magnets (made of N52) $B_r = 1.4$T.

The force per volume in the z direction (along the direction of the magnet's magnetisation) on the ferrofluid can be taken to be

$$\frac{f}{V} = \frac{\chi B_z \frac{dB_z}{dz}}{2\mu_0} \quad . \tag{2}$$

As this is tedious to calculate with the full field, you may use a dipolar approximation for the force on the ferrofluid in this part. Using this approximation the force per volume in the z direction is

$$\frac{f}{V} = -\frac{3\chi B_r^2 a^4 l^2}{8\mu_0 z^7} \tag{3}$$

Set up the stand so that the threaded rod through the aluminium tube has its flat face pointing downwards. You can rotate the aluminium tube to achieve this - make sure that the threaded rod does not hit the surface as you rotate the tube.

Stick the small bottle sideways to the top of the thick edge of the wedge using some of the blutack.

Magnet carrier in position for part A

Bottle setup for part A

A.1	Find the position of the large magnet where the ferrofluid is just able to be suspended in the clear unknown fluid and record the distance z on the answer sheet, along with its uncertainty. It can be difficult to achieve perfect balance, so if you are unable to keep the ferrofluid stably suspended, make the best estimate of the distance and its uncertainty. In the box on the answer sheet, draw a diagram showing how you measure the distance. Note that sometimes the ferrofluid may seem to 'split' into the magnetic blob and a separate, floating blob. This is usually due to the presence of a small amount of air in the bottle. You can use the magnet to adjust the position of the ferrofluid blob to find a clear space in the bottle to lift it.	0.8pt

A.2	Using your distance result and any other measurements as necessary, calculate the difference between the density of the ferrofluid and of the clear surrounding fluid, including its uncertainty.	0.8pt

Part B: Magnetic interaction: surface tension of a ferrofluid (1.2 points)

The ferrofluid moves under the influence of three energies: gravitational potential energy, surface energy associated with the surface tension, and the magnetic energy.

Using one of the magnets, observe what happens when you bring the magnet very close to the bottle. The spikes appear in the ferrofluid due to the normal field instability, which occurs when the effective frequency of gravity-capillary-magnetic waves in the fluid becomes imaginary.

The dispersion relation is

$$\omega^2 = \frac{gk\Delta\rho}{\rho_1+\rho_2} + \frac{\sigma k^3}{\rho_1+\rho_2} - \frac{k^2\mu_0 M_0^2}{1+(1+\chi)^{-1}}, \tag{4}$$

where σ is the surface tension of the ferrofluid at a ferrofluid-clear fluid interface, ρ_1 is the density of the ferrofluid, ρ_2 is the density of the clear fluid, $\Delta\rho = \rho_1 - \rho_2$, M_0 is the magnetisation in the ferrofluid, and k is the wavenumber.

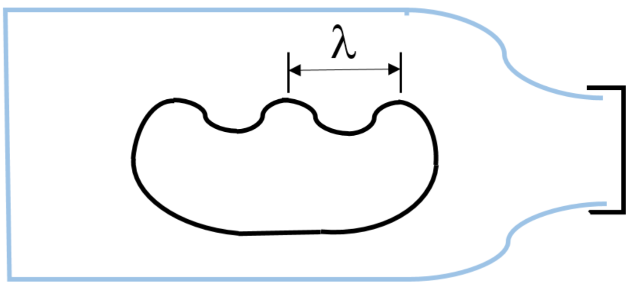
Applying the conditions $\omega^2 = 0$ and $\frac{\partial\omega^2}{\partial k} = 0$, the surface tension can be found in terms of the density difference and the effective wavelength of the perturbations to the fluid surface **at the point where the instability starts**. The relationship is

$$\sigma = \frac{g\Delta\rho\lambda^2}{4\pi^2}, \tag{5}$$

where λ is the distance between the centres of nearest neighbouring spikes when the magnetisation strength is at the instability threshold.

The magnet and carrier in position for part B.

Change the orientation of the threaded rod (rotating the tube while adjusting the rod position is the easiest way) so that you can raise and lower the magnet just under and through the hole in the top of the stand.



B.1	Using the glass bottle, take a measurement of the distance z_{crit} when the instability just starts to occur (peaks just start to form). Using the graticule or otherwise, measure the spacing of the peaks in the ferrofluid λ. at this position and record it. Estimate your uncertainties in both measurements. **Important hints:** if you hold the fluid strongly to the glass wall with a magnet, it will stick slightly to the glass, making your measurements more difficult. If this happens, you can use a magnet to draw the fluid away from the sticky area, and use another part of the bottle for this measurement.	0.6pt

B.2	Hence find the surface tension of the ferrofluid under the clear fluid, and its uncertainty.	0.6pt

Part C: Optical surface characterisation: non-spiking regime (4.1 points)

When the magnetic field is applied below the critical field, the surface of the fluid deforms to a first approximation as a balance between the magnetic and gravitational potential. In this next part we take the magnet to be sufficiently far from the fluid that it is well approximated by a dipolar field.

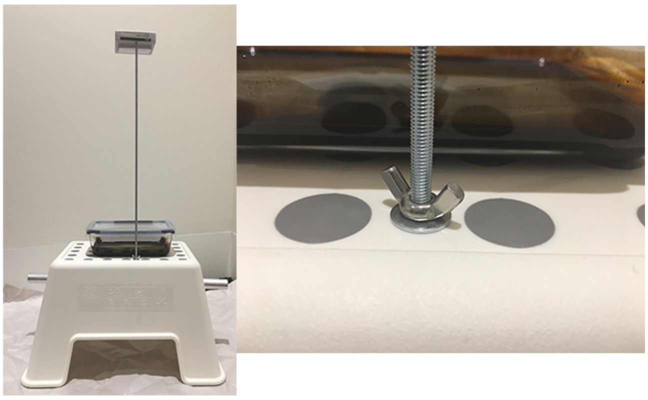
The surface deformation can be probed optically, using the surface of the fluid as an approximate spherical mirror. The effective radius of curvature of the centre of the deformation follows a power law, $R = \alpha z^n$ where α is a constant dependent on the materials and z is the distance of the magnet from the unperturbed surface of the fluid.

First we need to calibrate the magnetic threaded rod.

C.1	Use your equipment to find as precisely as you can the change in z for one turn of the threaded rod magnet carrier, including an uncertainty estimate. Record the measurements you use and draw a diagram of your setup.	0.6pt

Now take the glass dish of ferrofluid and place it on the stand, so that the hole in the centre of the stand is under the middle of the dish. *Do NOT open the sealed lid on the dish at any time during the experiment.* Load one of the small magnets onto the rod.

Thread the lamp onto the rod, and use the washer and wing nut to fix the threaded rod to the hole in the stand (see figure).

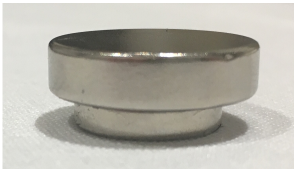


C.2	Using the relationship between the radius of curvature R and image magnification M for a spherical mirror, $R = \frac{2lM}{1-M}$ where l is the distance to the object, take measurements and then plot an appropriate graph to determine the constant n in the relationship above. Estimate the uncertainty in your answer.	3.5pt

Part D: Spiked surface characterisation: spike formation and disappearance (3.1 points)

For larger fields the surface will undergo the instability and form spikes as observed with the glass bottle.

D.1 Using a stack of one small and one large magnet, and the instability theory giving $\sigma = \frac{g\rho\lambda^2}{4\pi^2}$, determine the surface tension of the ferrofluid at an air interface. 0.5pt



D.2 Start with the magnet sufficiently far from the surface that no spikes are evident. Bring the magnet closer to the fluid and take distance readings for the appearance of each spike as it forms. Take another set of data as you withdraw the magnet, recording the position of the disappearance of each spike. Estimate the uncertainties in your measurements. 1.0pt

D.3 Plot a graph of the number of visible spikes against magnet distance z. Fit curves to your graph indicating clearly which direction the magnet was moving. 1.0pt

D.4 As the magnet's distance from the fluid surface changes, the fluid's gravitational, magnetic and surface energies change. Qualitatively sketch, as a function of the distance of the magnet from the fluid surface, the energy stored in surface energy and in magnetic potential energy. Mark any critical points from your previous graph and make clear the overall trend. You don't need to use the entire range of your data, just enough to show the idea. 0.6pt

Wave pulses in a magnetically active fluid (10 points)

Introduction

In this question you will investigate the properties of wave pulses in ferrofluids. The surface wave pulses which can be generated in the fluids are due to the interaction of the effects of gravity, surface tension and magnetic forces.

For wave pulses, there is no single wavelength or frequency of the pulse, however, it is known that the speed of propagation of a pulse in the absence of a magnetic field is

$$v(d) = \kappa\sqrt{d}, \tag{1}$$

where d is the depth of the fluid and α is a constant of proportionality. This relationship only applies for smaller wave amplitudes. When the wave amplitude is large the changed fluid depth due to the pulse causes changes in speed of propagation resulting in nonlinear effects. When the nonlinear effects are not significant, wave pulses approximately maintain their profiles as they propagate. However, when nonlinear effects are significant the pulses can change their profile as they propagate and often develop ripples either ahead or behind of the main pulse.

Part A: Plane pulses (1.3 points)

(a) (b) (c)

Photos of the required set up, (a) the glass dish with ferrofluid on the wooden base, (b) the glass dish on the wooden base, under the light shroud and stand, and (c) the camera on the light shroud and stand.

In order to generate plane waves in your ferrofluid, place the container with the ferrofluid on the wooden base, with a long side of the dish along the barrier at the edge of the base. The barrier is to ensure that your dish moves only in one direction when pushed. Ensure that the base is flat and level. To generate pulses, push the dish sharply with your hand a distance of around 2 cm. To ensure consistency in your pulse generation, you can use blu-tack and/or wooden spacers on the wooden base to mark starting and finishing positions for the ferrofluid container. It is recommended that you practice generating pulses at least a few times to be able to generate more consistent pulses.

The stand should then be set up over the dish. The lamp must be used to provide adequate lighting to be able to observe pulses in the fluid. Ensure that you can generate plane wave pulses and observe them by eye through the stand. Once you are able to do this, place the camera on the stand and make a video of a pulse. Note that the lighting and pulse amplitude may need to be adjusted to make a useful video. To adjust the lighting you can use blu-tack to place the light in an appropriate position, and may wish to use the aluminium foil to block or further direct the light.

When adjusting the lighting it is important to consider how to minimise light directly from the source into the camera, and also how to minimise reflections from the lid of the container. Reminder: you must NOT remove the lid from the container for safety reasons. Two options which can be useful in different situations are to have very diffuse light from all directions within the stand, or to direct light onto the fluid through the glass sides of the container.

See the description of the camera in Appendix A for how to make and playback videos. Note that the default automatic settings of the camera are suitable for making the required videos. If you are having difficulty clearly recording the observed effects, you need to adjust the lighting to be able to more clearly observe the wave pulses.

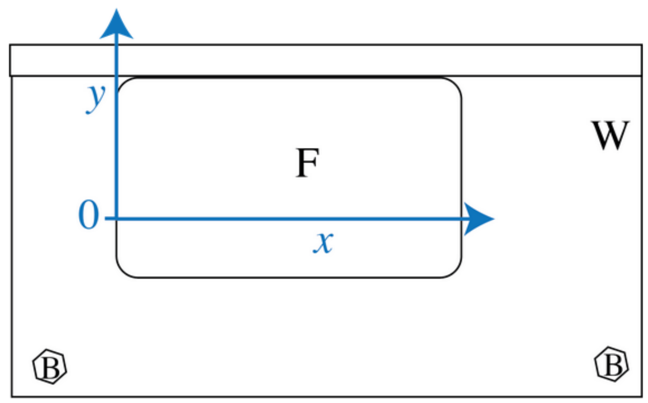
A.1	Draw a diagram of your set up, showing in particular how you positioned and directed the light.	0.3pt

A.2	Perform measurements and calculations from a video to find the speed of the waves in the ferrofluid. When performing measurements, also sketch diagrams of key frames of videos, showing important features, and marking any measurements made.	0.8pt

A.3	Make an estimate of the uncertainty of your measurement, showing any formulae you use to make the estimate.	0.2pt

Part B: Waves pulses in fluid of varying depth (3.4 points)

Insert the nylon bolts into the holes in the corner of the wooden base. By adjusting the bolts you can set up the platform so that the fluid depth in the container varies linearly with the distance from the lower edge (y-direction), and does not vary in the perpendicular direction (x-direction). We chose $y = 0$ to be where the depth of the fluid $d = 0$, so that the xy plane is the plane of the fluid's surface.



The set up of the glass dish with ferrofluid F on wooden base W so that the depth varies in the y-direction.

As the speed of waves in a fluid varies with depth, if a plane pulse is generated at one end of the container it will become curved as it propagates.

Set up your glass dish with ferrofluid on the wooden base so that you will best be able to characterise the variation of pulse speed with depth in the ferrofluid.

B.1	i. Draw a diagram of the container with ferrofluid as you set it up. Mark measurements, with uncertainties, of all distances which are important for you to know depth as a function of y in the container. ii. Write an expression for $d(y)$.	0.3pt

B.2	Generate plane wave pulses in your container, and qualitatively sketch a pulse you observe. Mark on your sketch the region where the pulse travels fastest with the letter A, and where it travels slowest with the letter B. Note: You will need to adjust the position of the light to clearly observe the pulses. To adjust the lighting you can use blu-tack to place the light in an appropriate position, and may wish to use the aluminium foil to make light blockers or light reflectors to further direct the light.	0.3pt

To a first approximation, we consider the pulse which is initially travelling in the x direction to remain travelling in the x direction only. Note: (x, y) are coordinates of points at time t are coordinates of points on the pulse at time t.

B.3	i. For a planar pulse which is at $x = 0$ when $t = 0$, find a relationship between x, y and t. ii. Mark on your diagram from B2, with the letter V, where within the glass dish with ferrofluid as you have it set up, this approximation is more valid?	0.3pt

B.4	Draw a diagram of your set up, showing in particular the position and direction of the light. Collect data to find κ and record in the table in the answer sheet. Include estimates of uncertainty for all data. When performing measurements, also sketch diagrams of key frames of videos, showing important features, and marking any measurements made.	1.2pt

B.5	i. Use a graphical method to calculate a value for κ. Include error bars for each measurement. Provide details of any formulae you use to calculate v, its estimated uncertainty Δv, d and its estimated uncertainty Δd. Record any additional calculated values for your graphs in the tables for B4. ii. From your data, state conditions on x, y and t for the relationship you developed in B3 to describe the observed wave pulses.	1.3pt

Part C: Wave and magnetic effects (1.8 points)

Warning: if the magnets collide they are likely to break. Broken magnets are dangerous, and yours will not be replaced if you break it.

As you observed in Question 1, in the presence of a magnetic field, the ferrofluid moves into the region of the strongest field. Your aim in this part is to investigate the interaction of wave a magnetic effects qualitatively.

Set up the ferrofluid container so that the ferrofluid has constant depth, as when you generated plane pulses. Next, place two small magnets underneath, as shown below. The magnets should have their poles aligned so that they repel each other when they are pushed together sideways in the channel on the wooden board. They should be positioned so that, with the ferrofluid in place, they are as close together as possible without repelling.

A safe way to put the magnets in place is to put **one** in the channel, then place the glass dish with ferrofluid over the track.You can use a wooden spacer in the track to position your magnet. The second magnet can then be slid into position, starting next to the dish, and using the spacer to push it along the channel under the ferrofluid.

Generate wave pulses using the following three methods:

- magnetically by rapidly withdrawing the large N42 magnet from near to the ferrofluid,
- mechanically by sliding the container on the wooden base, and
- mechanically by sliding the wooden base with the container fixed in place on top.

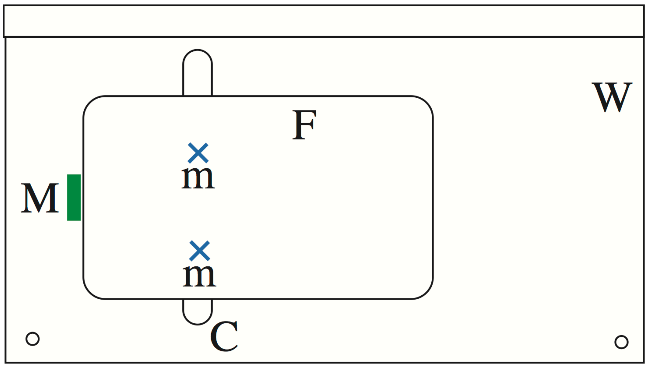


Diagram showing where to place the large magnet M, before rapidly removing it to generate a pulse travelling through the ferrofluid, where F is the glass dish with ferrofluid, m are the small magnets, W is the wooden base and C is the channel.

C.1	Sketch qualitative observations of the wave pulses for all three types of driving. Ensure that the diagrams clearly show how the presence of the magnets under the ferrofluid affects the wave front of the pulses. Identify all types of wave phenomena you observe. Mark with the corresponding number which effects cause which features of the observed pulse propagation. 1. reflection 2. refraction 3. doppler effect 4. beats 5. diffraction 6. interference (a) standing waves (b) from two slits or sources (c) from a diffraction grating (d) other	1.8pt

Part D: Internal properties of ferrofluid within a strong magnetic field (3.5 points)

In this part your aim is to quantify the effect of a magnet on pulse propagation in a ferrofluid.

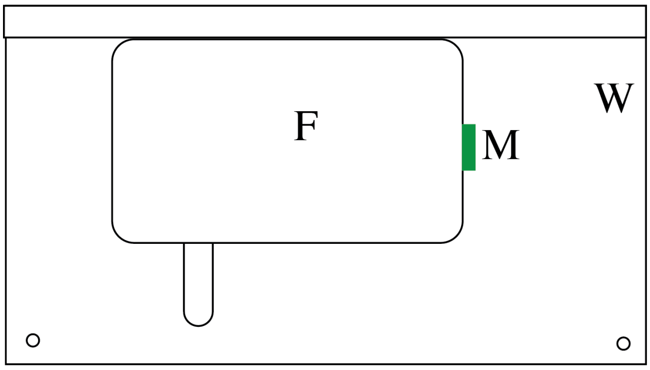


Diagram showing positioning of magnet in order to drive wave pulses in D1.

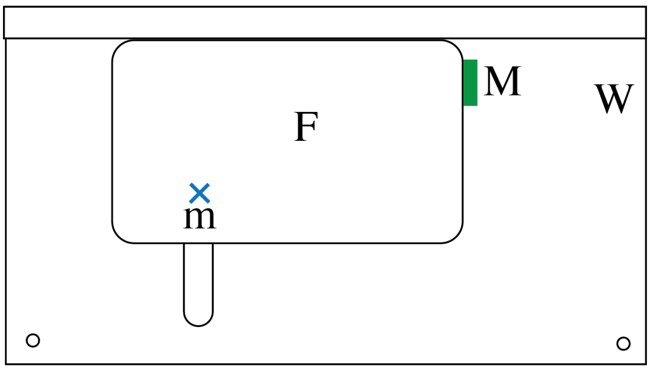
Set up the ferrofluid container so that the ferrofluid has constant depth, as when you generated plane pulses.

Generate pulses magnetically using the **large** N42 magnet, as in C1, and adjust the lighting to clearly observe the magnetically driven pulses.

D.1	Qualitative sketch a magnetically driven pulse in the ferrofluid.	0.2pt

D.2	Determine the speed of the pulse with its uncertainty. Sketch and diagram of your set up, including light position and direction, and record all data and formulae you use. When performing measurements, also sketch diagrams of key frames of videos, showing important features, and marking any measurements made.	0.8pt

Now place a single small magnet into the channel in the wooden base, and then place the dish and ferrofluid over the magnet. Its position can be adjusted using the wooden spacers, both in the track and to push the magnet around under the glass dish.



The position of the small magnet under the glass dish with ferrofluid is marked with an x, and the position of the large magnet, which is held outside the dish and then rapidly removed to generate a wave pulse, is labelled M.

Generate wave pulses magnetically, by starting with the large magnet in the position marked in the diagram above, and observe them travelling through the ferrofluid, including the region with the strong magnetic field.

D.3	Sketch qualitative observations of the wave pulses. Ensure that the diagrams clearly show how the presence of the magnet under the fluid affects the wave front.	0.4pt

D.4	Draw a diagram of your set up to determine the effect of the presence of the magnet under the fluid on the propagation time for a wave pulse. Clearly mark the positions of the magnets and also the light.	0.3pt

D.5	Make and record measurements with uncertainty estimates to determine the travel time of a pulse across the region with the magnet. When performing measurements, also sketch diagrams of key frames of videos, showing important features, and marking any measurements made.	1.0pt

D.6	Does the additional depth of the ferrofluid over the magnet explain the difference in speed of the wave pulses? Show calculations to support your answer.	0.8pt

Appendix A: How to use Canon IXUS-185

To turn the camera on or off press the button (O) labeled "ON/OFF" at the top of the camera. Note that the camera automatically turns off after a time.

To focus the camera, press the button (S) on the top of the camera part way.

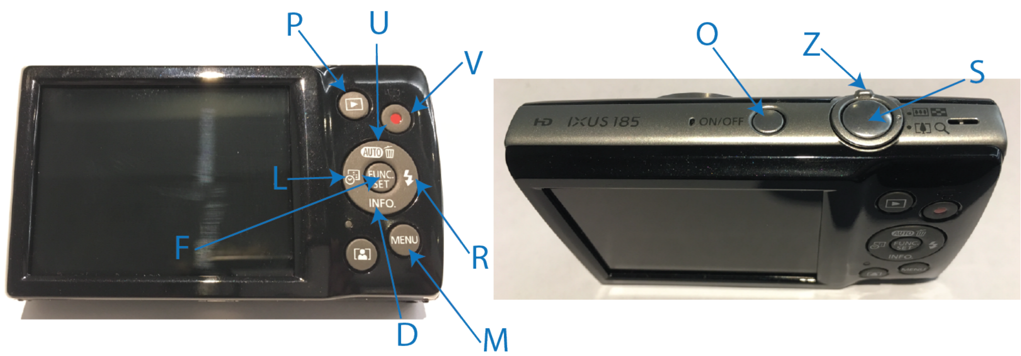
To take a photo press the button (S) on the top of the camera.

To take a video press the button marked with a red dot (V) to start recording. Press it again to stop recording. Note that the default HD videos record at 25 frames per second.

To view photos or videos you have taken, press the review button (P). Use the right (R) and left (L) buttons on the ring button on the back of the camera to navigate through your photos and videos.

- You can zoom in or out of photos using the ring (Z) on the top of the camera. Note: you cannot zoom after taking a video.
- To play a video press the "FUNC. SET" button (F) in the middle of the ring button on the back of the camera twice.
- To pause a video, press the "FUNC. SET" button (F) in the middle of the ring button on the back of the camera while the video is playing.
- When a video is paused you have options on the bottom of the screen. From left to right the options are: Exit, Play, Slow Motion, Skip Backwards, Previous Frame, Next Frame, Skip Forward. Use the right (R) and left (L) buttons on the ring button on the back of the camera to navigate through options, and the "FUNC. SET" button (F) in the middle of the ring button on the back of the camera to change the volume of video playback.

To zoom in or out, push the ring (Z) to the left to zoom out and to the right to zoom in.



To enter the camera settings menu press the button (M) labeled "MENU", then use the right ring button (R) on the back of the camera to enter the camera settings menu. Note that the first option is to mute the camera sounds.

- Use the right (R), left (L), up (U) and down (D) buttons on the ring button on the back of the camera to navigate through the menu.
- Use the "FUNC. SET" button (F) in the middle of the ring button on the back of the camera to select options.
- Use the "MENU" button (M) to exit the menu or to move back up through the menus.

To change the language of the camera, first enter the camera settings menu (see above). Press the down right button (D), labelled "INFO." on the back of the camera to scroll down the many page list to find "Language". Next press the right ring button (R) on the back of the camera to enter the list of language options. The the right (R), left (L), up (U) and down (D) buttons on the ring button on the back of the camera to navigate to the language of your choice. Use the "FUNC. SET" button (F) in the middle of the ring button on the back of the camera to select the language.

Note: You are not expected to change any settings on the camera, except for optionally changing the language, as the default automatic settings are suitable for the videos are you are required to make. You will benefit more from adjusting the lighting of your set up.