General Instructions: Experimental Examination (20points)

May 25, 2023

The experimental examination lasts for 5 hours and is worth a total of 20 points.

Before exam

- You must not open the envelopes containing the problems before the sound signal indicating the beginning of the competition.
- The beginning and end of the examination will be indicated by a sound signal. There will be announcements every hour indicating the elapsed time, as well as fifteen minutes before the end of the examination (before the final sound signal).

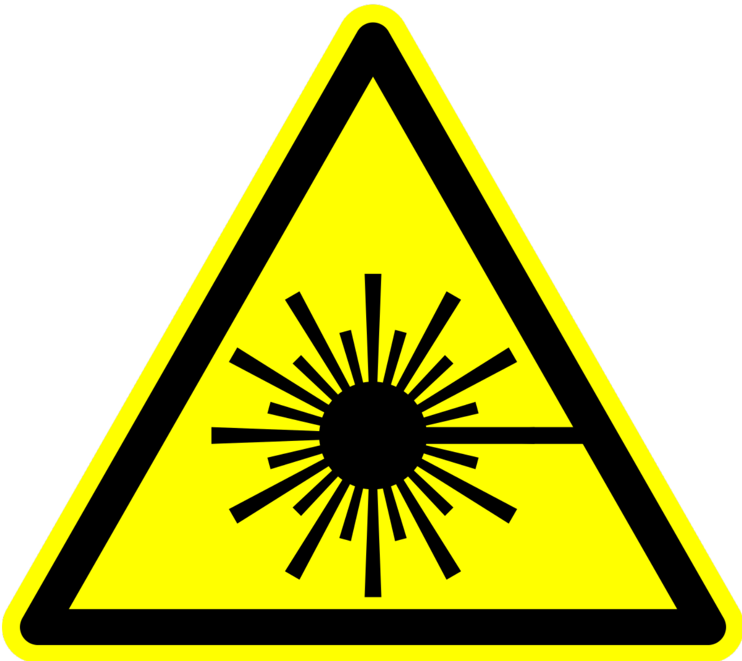
During the exam

- Dedicated answer sheets are provided for writing your answers. Enter the observations into the appropriate tables, boxes or graphs in the corresponding answer sheet (marked A). For every problem, there are extra blank work sheets for carrying out detailed work (marked W). Be sure to always use the work sheets that belong to the problem you are currently working on (check the problem number in the header). If you have written something on any sheet which you do not want to be graded, cross it out. Only use to front side of every page.
- In your answers, try to be as concise as possible: use equations, logical operators and sketches to illustrate your thoughts whenever possible. Avoid the use of long sentences.
- **Explicit error calculation is not required unless explicitly asked for.** However, you are asked to give an appropriate number of significant digits when stating numbers. Also, you should decide on the appropriate number of data points or measurement repetitions unless specific instructions are given.
- You may often be able to solve later parts of a problem without having solved the previous ones.
- You are not allowed to leave your working place without permission. If you need any assistance (need to refill your drinking water bottle, broken calculator, need to visit a restroom, etc), please draw the attention of a team guide by putting one of the two flags into the holder attached to your cubicle ("Refill my water bottle, please", "I need to go to the toilet, please", or "I need help, please" in all other cases).

At the end of the exam

- At the end of the examination you must stop writing immediately.
- For every problem, sort the corresponding sheets in the following order: General instructions(G), questions (Q), answer sheets (A), work sheets (W).
- Put all the sheets belonging to one problem into the same envelope. Also put the general instructions (G) into the remaining separate envelope. Make sure your student code is visible in the viewing window of each envelope. Also hand in empty sheets. You are not allowed to take any sheets of paper out of the examination area.
- Wait at your table until your envelopes are collected. Once all envelopes are collected your guide will escort you out of the examination area. Take your writing equipment bag with you and hand it in at the exit. Also take your water bottle with you.

Caution



LASER RADIATION HAZARD
BE AWARE OF THE LASER BEAM.
DO NOT STARE INTO THE BEAM.
AVOID DIRECT EXPOSURE OF THE EYES TO THE LASER.

Instructions of oscilloscope alignment Siglent SDS 1152CML+

1. First to acquire waveform on the screen connect probe's BNC connector (A) to the CH1 input and probe's hook (B) to the 1 KHz square wave output pin. Then press the [Auto] button, oscilloscope will acquire the waveform automatically (Fig.1).

Figure 1

2. Remove hook (B) from the test output pin and connect to CH1 pin of the Experimental setup control box shown in Fig. 2

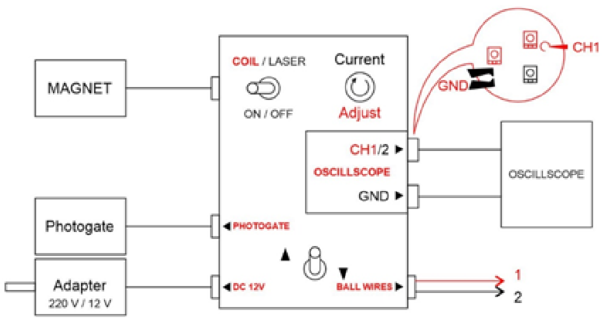


Figure 2

3. Match Trigger axis (labelled by white T on the top of screen) to horizontal axis by Level knob.

4. Check experimental conditions by pressing [Trigger Menu]. It should be same as shown in Fig. 3.

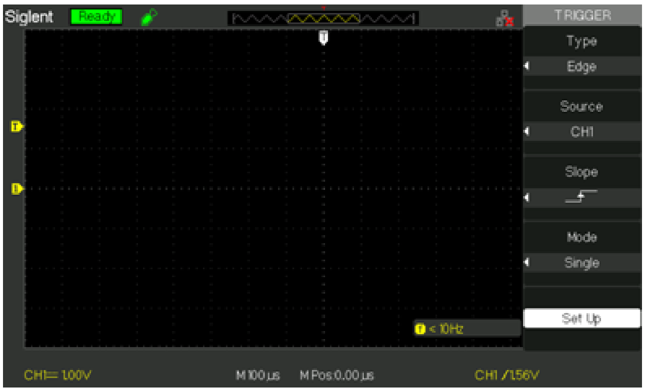


Figure 3

5. Voltage of Channel1 (CH1) is adjusted to 2 V by knob Variable in Vertical of CH1.

6. The scale should be adjusted to 250 ms by Zoom knob in Horizontal.

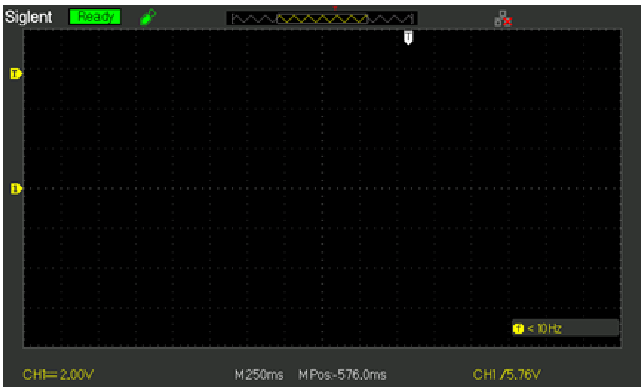


Figure 4

7. Press on Run/Stop button after attaching the ball onto electro-magnet. The color of the button will turn green from red.

8. "Ready" in green will appear on the upper-left corner of the screen. 9. Start the measurement by releasing the ball from electro-magnet.

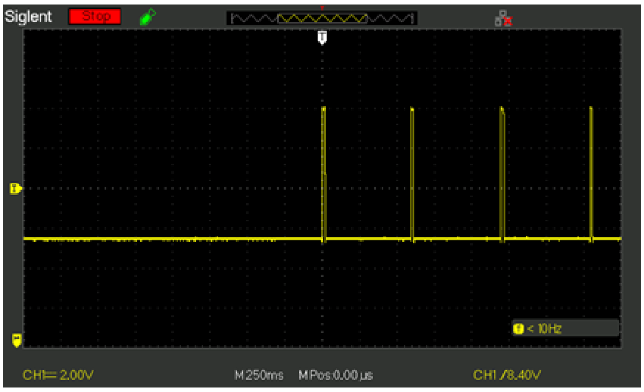


Figure 5

10. Obtain Δt by measuring the wight of 3rd pulse. 11. Measure the wight of 3rd pulse with 7 significant figures. When you try to rotate (Horizontal position) knob you can see this value on the left down corner of the screen M Pos =1.195550s.

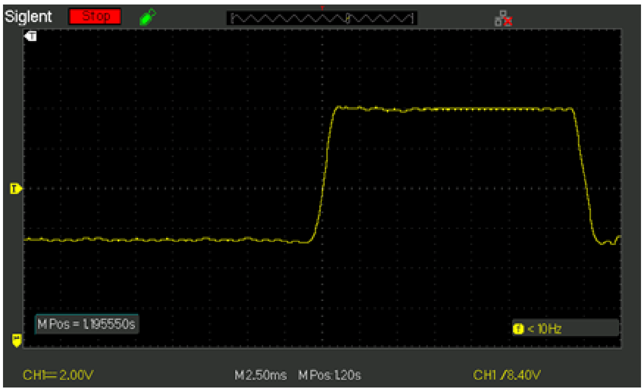


Figure 6

White T mark is know on the left top side of the screen, and pointing to the left means trigger position is outside of the screen and the time to the trigger position from the screen center is measured with 7 SF. 12. For oscillation experiment you should not use the first pulses, because this signal is affected by magnetization and wire tension. Therefore 2nd and 4th pulses or 3rd and 5th pulses must be used to determine the oscillation period or speed measurement. To make time measurement for nth pulse you put time division to 250 ms range, then you will receive the following measurement. Then using horizontal position knob bring the nth pulse at the center of display. Then change time division to smaller value to widen the pulse shape. To measure the pulse width, you must bring rising edge's half amplitude at the center of the display. Rotate (Horizontal position) knob to see precision time value on the display. Then increase time value and bring half amplitude value of the falling edge at center of display, and repeat precision time measurement. Difference of these two values will be very precise pulse width value.

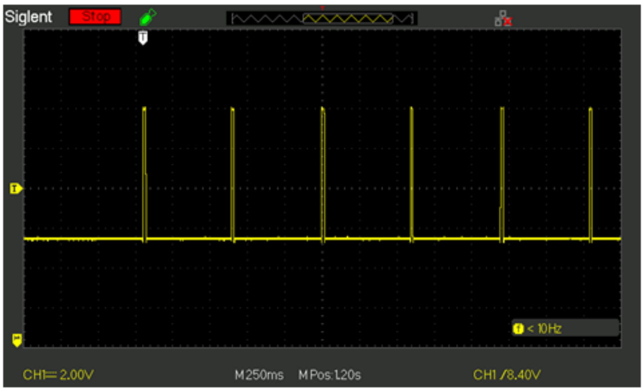


Figure 7

Oscilloscope Siglent SDS 1202X-E

1. First exercise same with previous model oscilloscope. 2. Connect photo gate to the oscilloscope input and swing the ball. 3. Then press blue [Auto setup) button, the following screen will occur.

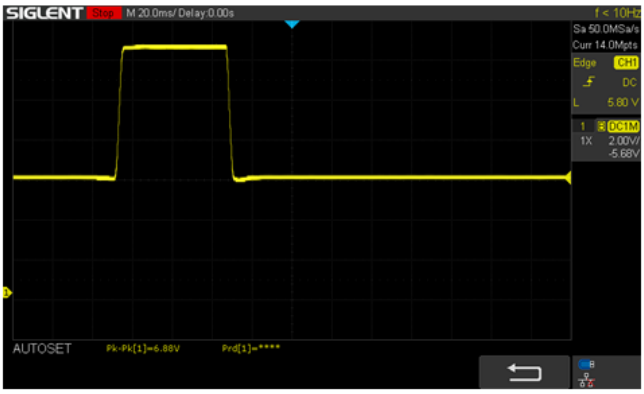


Figure 8

The pulse shape may be unstable. Just turn Time division knob to the 20 ms. When pulse occurs on the screen just press (Run/Stop) button to froze the signal. 4. By using vertical position knob and Volts/division knob put the signal at the center of the screen.

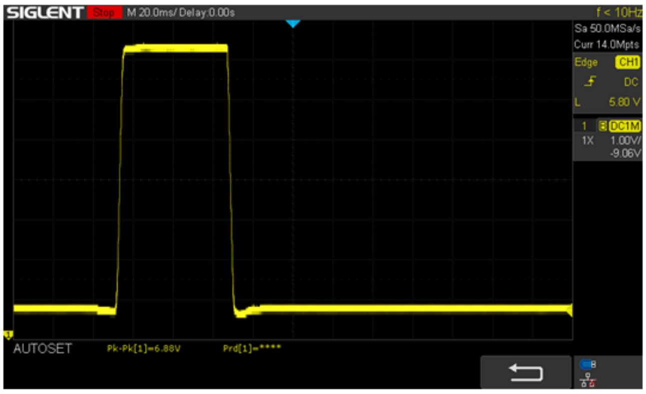


Figure 9

5. Change Time division value to 500 ms. Hold the ball with magnetic holder and press [Single) button, screen will be blank, then release the ball from the holder switching off the control box. The following photogate pulse will be recorded in the oscilloscope memory.

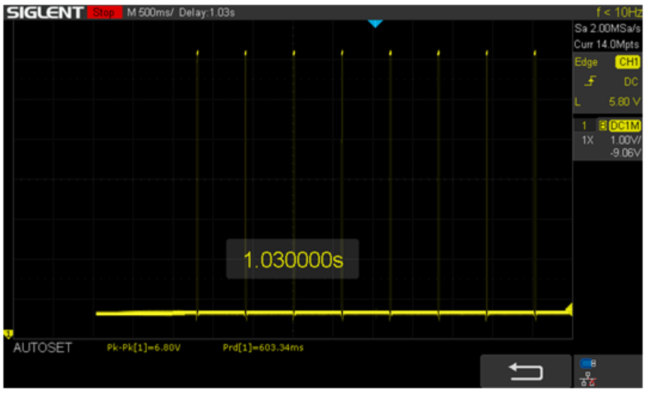


Figure 10

Note that blue triangle at the top center of the screen. This is the time origin at zero. By rotating (Horizontal position knob) you can place third pulse at the center. Second and 4th pulses can be used as well. Don't use first pulse. It has magnetic holder and wire tension influence.

6. When you change Time division value to 100 us and by turning Horizontal position place the signal rising edge at the exact center of the screen. In the middle of the screen you will receive precise value of time difference from the origin.

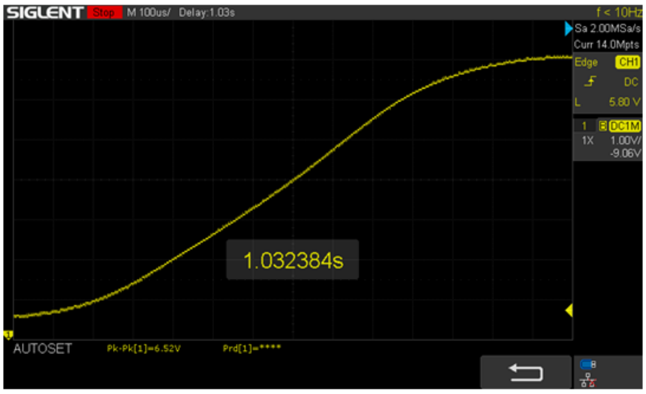
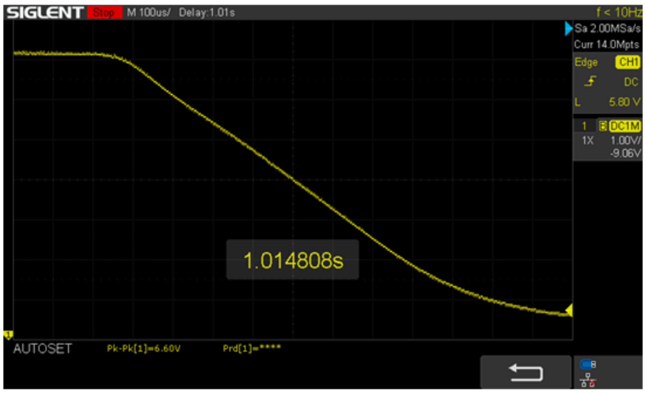


Figure 11

7. Change time division to the 20 ms back, then rotate horizontal position to bring falling edge of the pulse. Then change Time division to back 100 us and position half amplitude at the center of the screen, you will receive another precise value of time. So difference of the these times will be pulse width.



8. Then you can choose 5th pulse rising edge again. Now you able to find the exact period of the oscillation from this measurement.

Hertzian Contact Stress [10 points]

Please read the general instructions before you start this problem.

Introduction

The Hertzian contact theory is a classical theory of contact mechanics and is a very useful tool for engineers and researchers. Even though the derivation of the theory is relatively difficult, the final solution is a set of simple analytical equations relating the properties of the system to the developed stress. Hertz contact theory is derived from the analytical solution of elasticity theory equations under the so called half-space approximation:

1. One of the surfaces is a half-space (one of two regions formed when a plane divides 3D Euclidean space) of infinite extend (Figure 1).
2. Pressure profile is parabolic (Equation 2)
3. This infinite half-space model can be used for two spherical balls with the same or different sizes.

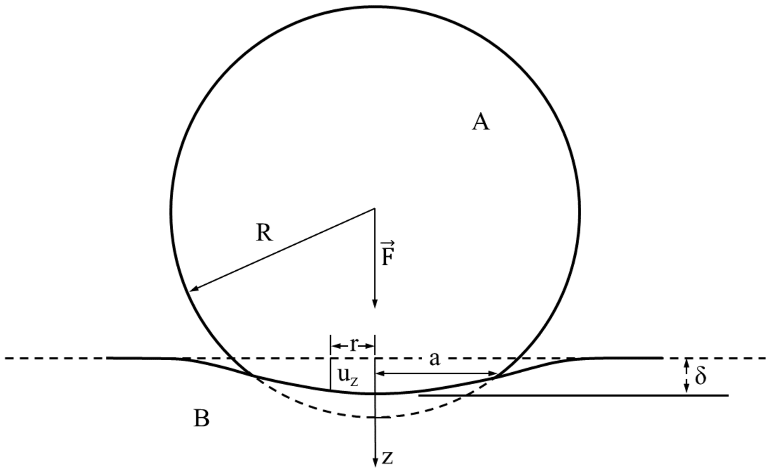


Figure 1 (A - Spherical ball, B - Infinitely large half-space)

If there are only vertical forces acting on the surface, the elastic deflection of the surface under applied pressure is given by the following relation:

$$u_z(x,y) = \frac{2\pi}{E'} \iint \frac{p(x',y')}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy' \tag{1}$$

Here u_z is the elastic deflection, $\frac{1}{E'} = \frac{(1-v_1^2)}{E_1} + \frac{(1-v_2^2)}{E_2}$ is the reduced elastic modulus, v_1, E_1, v_2, E_2 are respectively the Poisson's ratio and Young's modulus (these values are constant for given balls and will be provided for calculations in task E.1) of the spheres, $p(x,y)$ is the contact pressure. If the pressure profile is arbitrary, this equation does not lead to the analytical solution. However, the Hertz solution is obtained under the assumption of a parabolic pressure distribution, which is a very good approximation for spherical, elliptical or cylindrical bodies in contact:

$$p(r) = p_0 \left(1 - \frac{r^2}{a^2}\right)^{1/2} \tag{2}$$

where r is the distance to the arbitrary point on the surface and a is a parameter known as the Hertz contact radius. Parameter p_0 is also called the maximum Hertz pressure. Substituting this into the Equation (1) for deflection leads to the following expression for the Hertzian pressure:

$$u_z = \frac{\pi p_0}{4E'a}(2a^2 - r^2), r \leq a \tag{3}$$

This experimental problem has two parts: large angle pendulum and two balls collisions.

General Safety Precautions:

1. Be sure to switch off the equipment before plug in/out its power cord. Damage may occur.
2. Do not change the oscilloscope settings unless instructed in the problem.
3. Be careful not to spill the drinking water onto nearby electronics and electrical power sockets.
4. Do not disassemble the first experimental setup, unless you have finished the experiment. You would not be able to reassemble again.

Apparatus

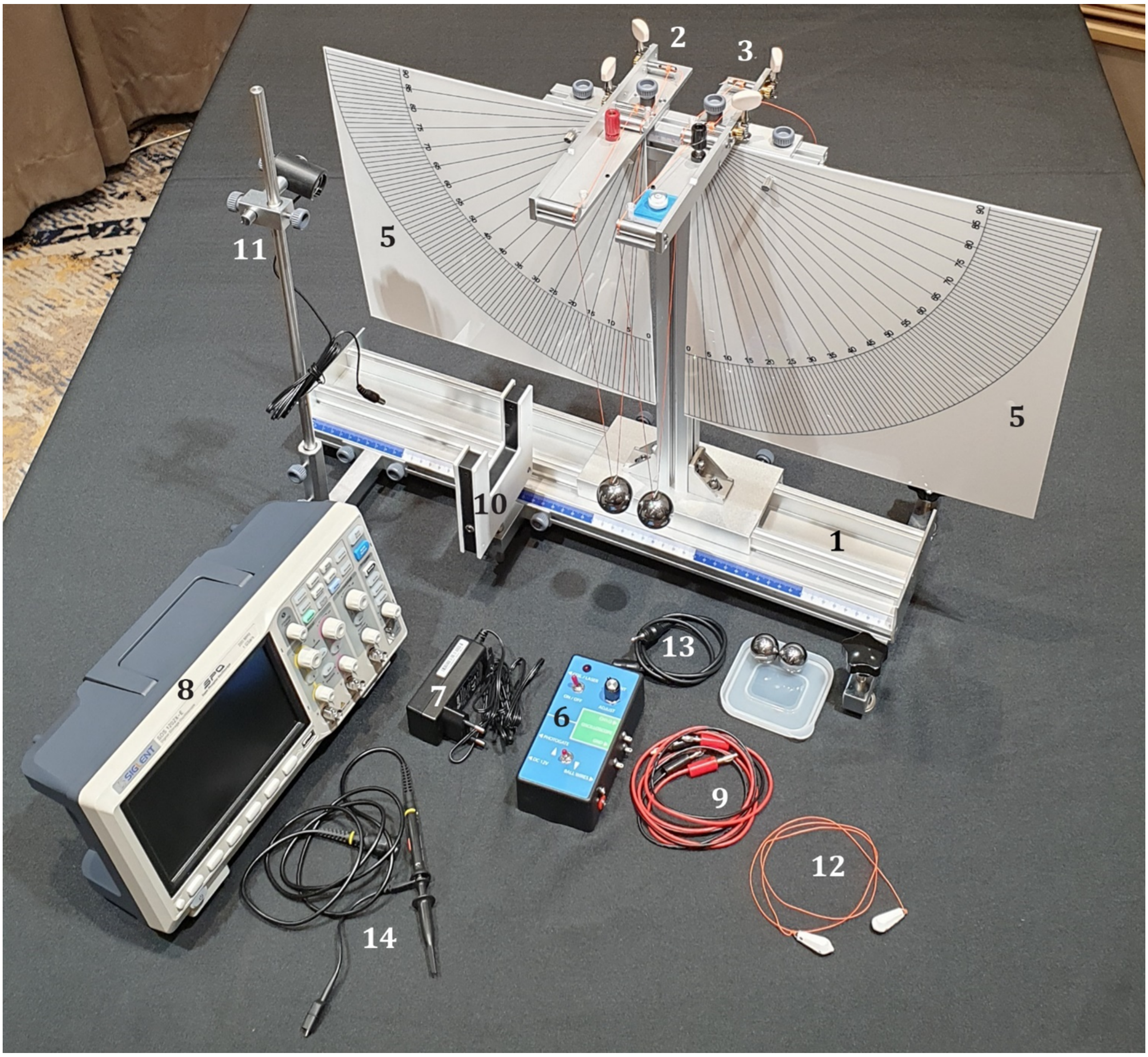


Figure 2

1. Base flatform of the setup. (Base will be used for both experiments)
2. Pendulum 1 with hanger unit (big ball is hung already).
3. Pendulum 2 with hanger unit (big ball is hung already).
4. Two small sized balls.
5. Angle measuring screens for Left and Right sides.
6. Electric junction box.

7. Power adaptor.
8. Oscilloscope with measuring probe 14.
9. Connecting wires for ball contact measurement.
10. Photogate for pendulum with connecting cable (13).
11. Electromagnetic ball holder with adjusting stand.
12. Wire with two plumb bobs.

Part A: Large angle pendulum [1.4 points]

In this part, you are required to determine the maximum speed and angular amplitude relation of the single ball pendulum starting from large angle to small angle, for as wide a range of angles as possible. To use the pendulum, you should understand the ball position adjusting techniques. There are two string tuning knobs on each pendulum hanger. They are used for adjusting ball positions up/down for z direction and forward/backward for y direction. To adjust ball positions left/right for x direction the pendulum hanger unit should be used (Figure 3).

For this task you are required to use only one of the pendulums. Therefore, please put one of the already fixed balls on the head of the screw for tightening pendulum hanger. Please pay attention to the pendulum wires to be not disturbing the movement of your working pendulum.

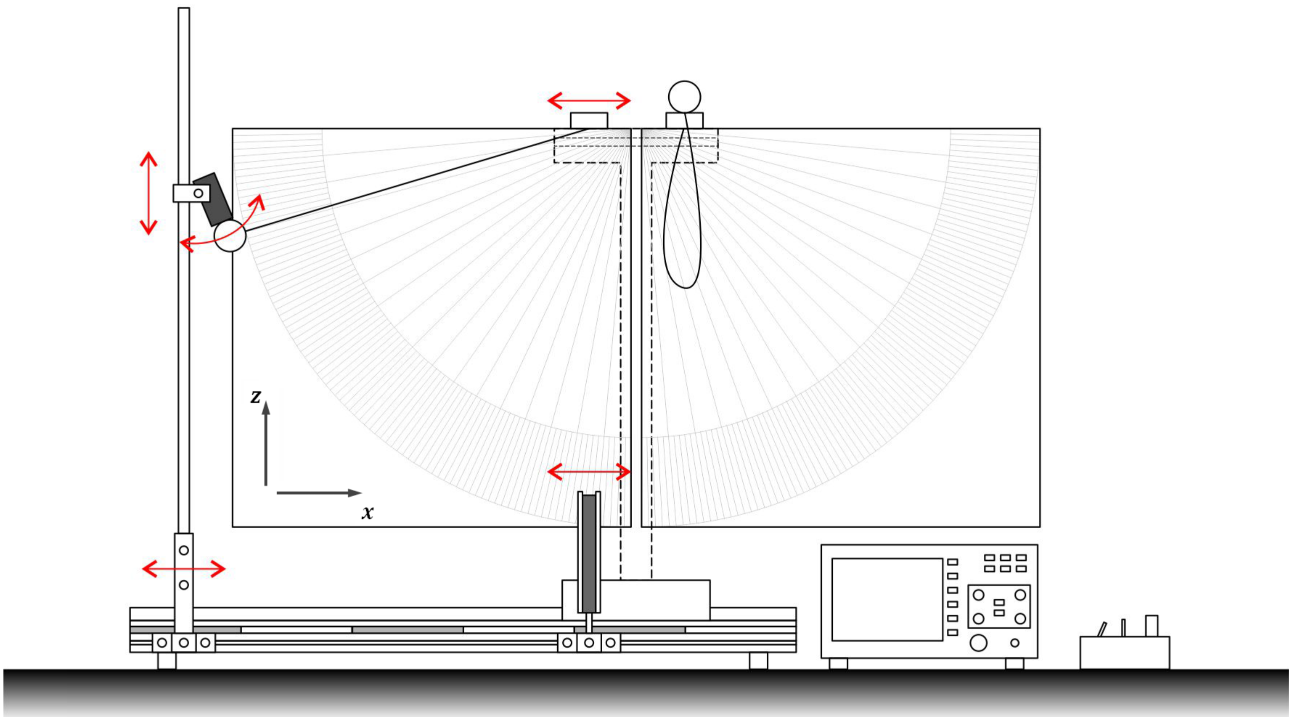


Figure 3

A.1	Find gradient (or slope) of Δt^{-1} against sin $\frac{\varphi_0}{2}$. Here φ_0 is the initial angle (or the angular amplitude) of the pendulum and Δt is the time required for the fall to pass through the photogate. Remark 1. To measure time precisely on Digital storage oscilloscope, you are advised to follow oscilloscope instructions in the General Instructions (G). Beware of the model of your oscilloscope and instruction exercises. To save time you can make measurement for task B.1 together with this task. When the ball is released from electromagnetic holder, wire elasticity and magnetization of the ball may affect the initial speed of the ball. Therefore, you must not choose the first signal from the photogate.	1.2pt

A.2	Derive the general equation to find maximum speed of the ball for any given initial angle of the pendulum.	0.2pt

Part B: Period of the oscillation [1.9 points]

The period of the pendulum depending on the angular amplitude is given by the following series.

$$T = T_0 \left(1 + \alpha \cdot \sin^2 \frac{\varphi_0}{2} + \beta \cdot \sin^4 \frac{\varphi_0}{2} + \cdots\right) \tag{4}$$

Here T_0 is the pendulum period when amplitude is small, α and β are constant coefficients, and φ_0 is the angular amplitude.

B.1	Plot the linearized graphs of $T = f(\varphi_0)$ by measuring period of the oscillation. Note that different linearization, for appropriate regions, are required. Determine T_0, α, β from the graphs. Remark 2. To measure period very precisely on Digital storage oscilloscope, you are advised to do oscilloscope using exercises on the end of the Question sheets. Beware of model of your oscilloscope and instruction exercises.	1.6pt

B.2	By using gradient found in task A.1 and T_0 in task B.1 find the exact value of gravitational acceleration of Ulaanbaatar. The steel ball's diameter is $d = 31.75$mm.	0.3pt

Part C: Behaviors of the collisions [0.7 points]

The initial positions and the conditions of the balls will strongly affect first collision and further collisions processes: position of collisions, contact time and contact positions on the balls. To execute this task please pay attention to the ball position adjusting techniques.

In this part you will observe the influence of the contact position on the behavior of the collisions of identically sized balls. Observe the behavior of the collisions for each of the scenarios described in sections C.1-C.4 (matching the appropriate description in Figure 4) and for each task match between the setup and a time dependence graph from Figure 5.

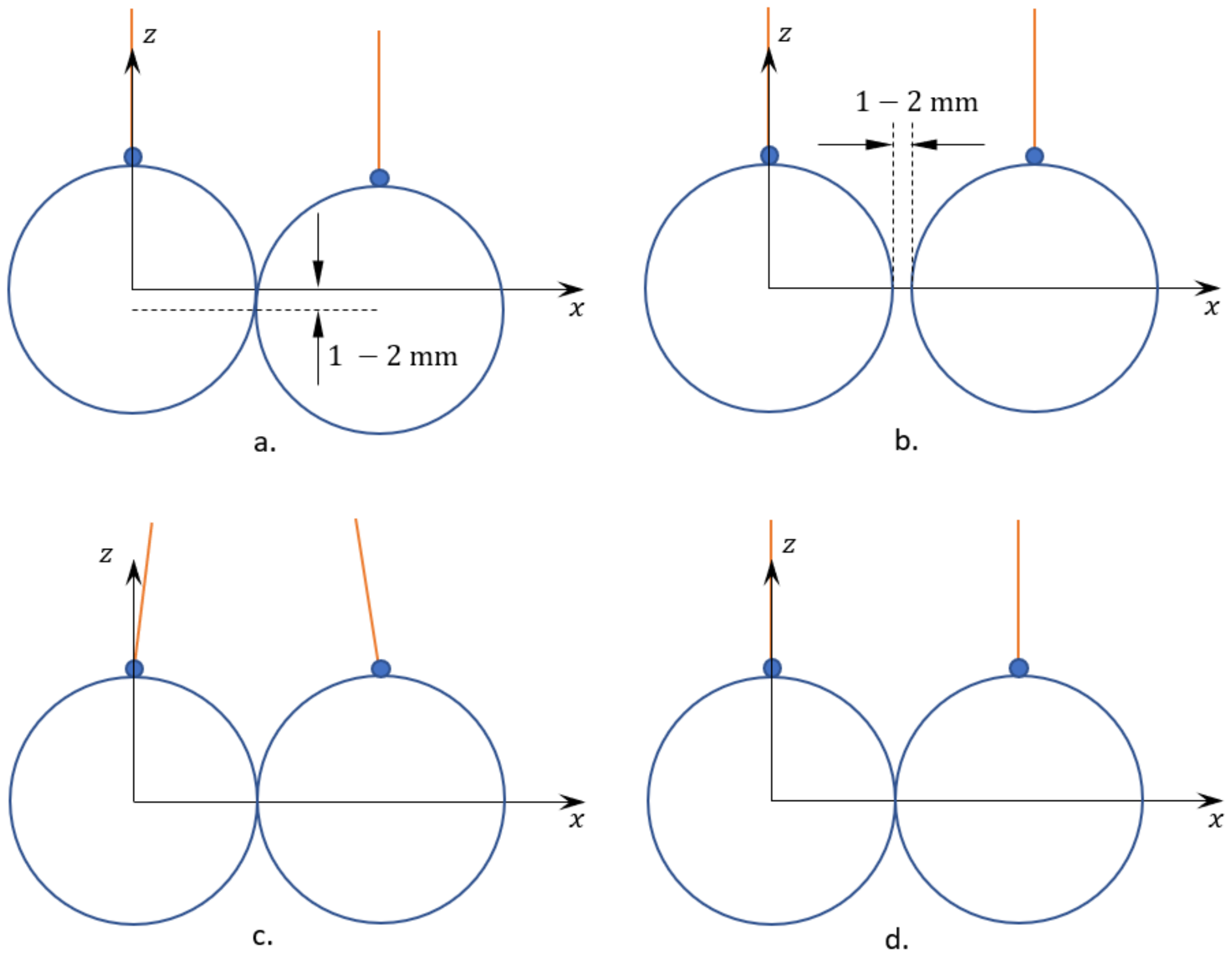


Figure 4

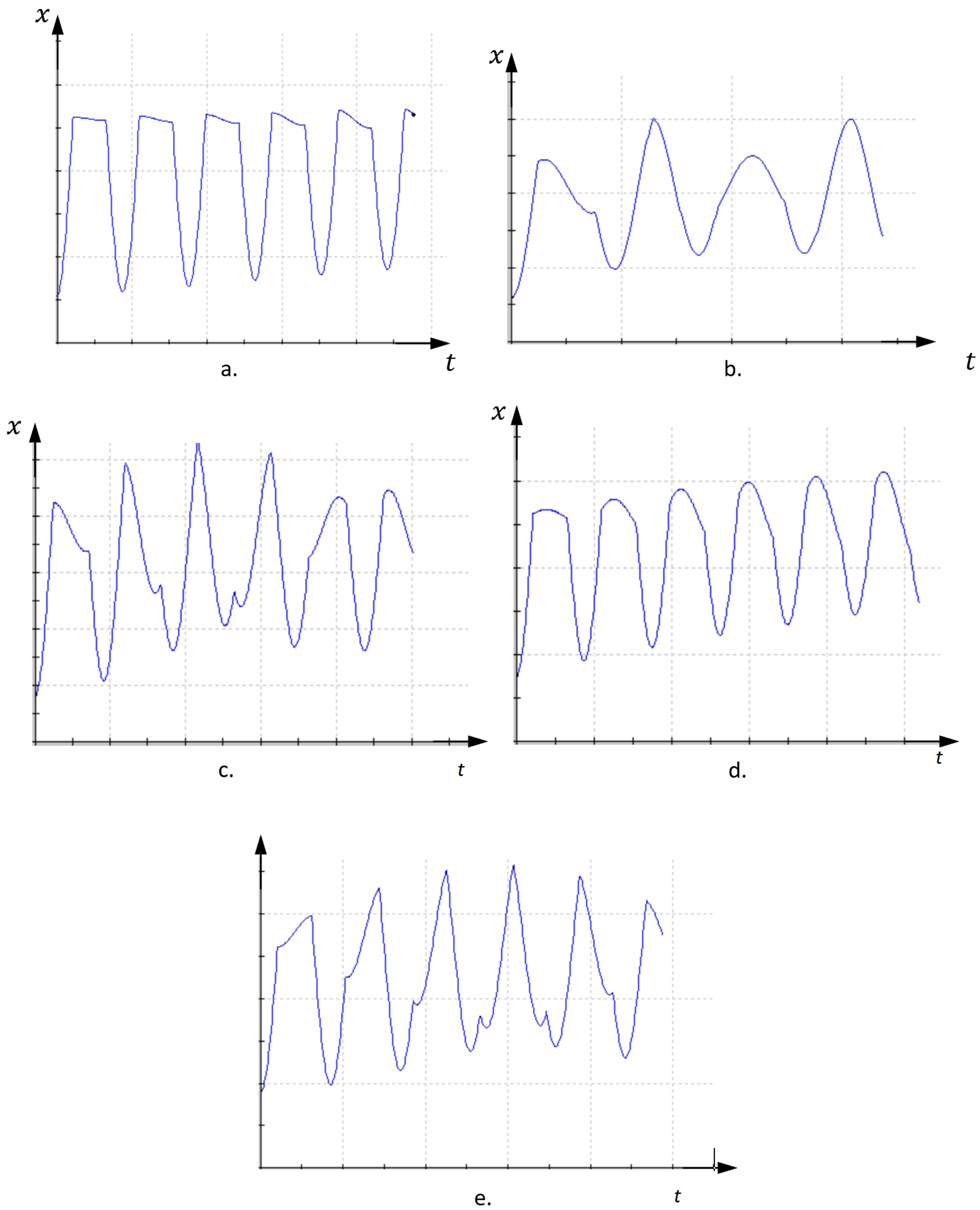


Figure 5

C.1	When two balls are hung and stable at the equilibrium position, we assume that the center of the first ball as the origin of the coordinate system. Then the positions can be described as: the displacement between two ball centers in x direction $d_x = 2R$, displacement in y direction $d_y = 0$, displacement in z direction $d_z \approx 1$ to $2\ mm$and the hanging wires are almost parallel (Figure 4.a). When ball 2 is in equilibrium position, ball 1 is released from angle from 25° to 30° without initial speed. Observe by eyes the influence of the contact position on the collision behavior of identically sized balls.	0.2pt

C.2	$d_x - 2R \approx 1\ mm$, $d_y = 0$, $d_z = 0$ and two wires are parallel. That means the ball hanger units should be moved away by $1mm$ distance (Figure 4.b) . When ball 2 is in equilibrium position, ball 1 is released from an angle between 25° to 30° without initial speed. Observe by eyes the influence of the contact position on the collision behavior of identically sized balls.	0.2pt

C.3	The two balls are in contact, $d_x \approx 2R$, $d_y = 0$, $d_z = 0$ and the ball hanger units touch each other. In this case wires are not parallel (Figure 4.c). When ball 2 is in equilibrium position, ball 1 is released from an angle between 25° to 30° without initial speed. Observe by eyes the influence of the contact position on the collision behavior of identically sized balls.	0.2pt

C.4	The two balls are in contact, $d_x \approx 2R$, $d_y = 0$, $d_z = 0$ and the ball hanger units are separate, the ball wires are parallel (Figure 4.d). When ball 2 is in equilibrium position, ball 1 is released from an angle between 25° to 30° without initial speed. Observe by eyes the influence of the contact position on the collision behavior of identically sized balls.	0.1pt

Part D: Time of collisions [3.0 points]

The Hertzian contact will last for a certain time and the balls will separate. In this part collision time (τ) dependence on which parameters of the balls will be studied by making several collisions between same and different sized balls. To measure the contact time the electric circuit is used which is shown in Figure 6. Pendulum nylon wires are only for oscillation and additional thin metal wires connected to the balls. When you change the wires please make sure wires contacting the balls securely. To maintain this please use provided wooden toothpicks to lock metal wires and nylon wires without any slide through the ball hanger slot. Do not block both sides of the slot! To change the wires, you need to remove the locking toothpick pin. For this action the paper clip will be provided (Figure 7).

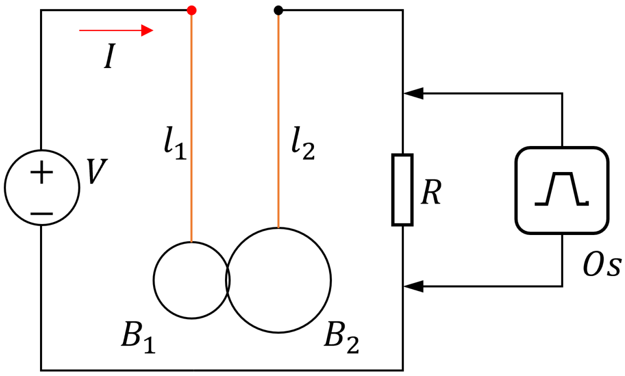


Figure 6 (B_1 - Ball 1, B_2 - Ball2, Os - Oscilloscope)

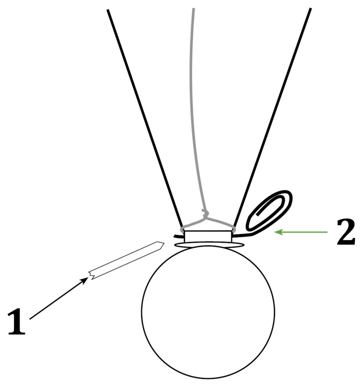


Figure 7 (1 - toothpick, 2 - paper clip)

D.1	The collision time τ between the balls is given by the equation: $\tau = A \cdot x_1^{e_1} \cdot x_2^{e_2} \dots \cdot x_n^{e_n}$ where A is a dimensionless constant, and $x_1, x_2, \dots, x_n$ (for some integer n) are physical parameters of the collision. Determine all of these physical parameters. The values of the exponents $e_1, e_2, \dots, e_n$ will also be determined in parts D.2 and D.3.	0.4pt
D.2	By experiment, plot appropriate linear graphs to find the values of some exponents in the expression for τ in D.1.	1.2pt
D.3	Using dimensional analysis and the values of the exponents obtained in D.2, find the values of the remaining exponents.	0.4pt
D.4	Find the numerical value of A with high accuracy (at least with 4 SF).	1.0pt

Part E: The parameters of Hertz deformation [3.0 points]

By using the equation of maximum speed in task A.2 and the experimental data based on task D.2, find the following parameters for a range of initial angles ($\nu = \nu_1 = \nu_2 = 0.3$, $E = E_1 = E_2 = 200$ GPa, $m_1 = 131.48$ g, $m_2 = 67.55$ g, $d_1 = 31.74$ mm, $d_2 = 25.42$ mm):

E.1	Find the expression and calculate the numerical value of average force F_{av}	0.6pt
E.2	Find the expression and calculate the numerical value of the Hertz deflection δ	0.6pt
E.3	Expression and value of the Hertz radius a	0.6pt
E.4	Find the expression and calculate the numerical value of the Hertz pressure P_0	0.6pt
E.5	Find the expression and calculate the numerical value of the average pressure P_{av}	0.6pt

Problem 2: Interference from thermally deformed surface (thermos-deformation)

When light interacts with matter, it gives rise to various phenomena. This experimental study focuses on investigating the diffraction of light from a light-deformed acrylic (PMMA) surface.

Acrylic is 5 times stronger than regular glass and is an organic material that can be used in a variety of applications. The liquid monomer of methyl methacrylate serves as the primary material for acrylic. The liquid state has lower density compared to the solid state. Various colorants and hardeners are added to the liquid monomer. The thermoforming temperature ranges from 160 to 190°C. Acrylic is an environmentally friendly material as it does not release any harmful chemicals. Therefore, it is commonly used in items such as children's toys, utensils, and equipment frames. Moreover, it is resistant to water.

Test equipment:

1. Laser with voltage adjustment tool

2. Screen with holes
3. Two multimeters (one amperemeter and one voltmeter)
4. Connecting wires for laser source and power control unit
5. Connecting wires for multimeters
6. Control box
7. Power supply adaptor
8. Acrylic (PMMA) target with adjustable stand

Schematics of the experimental setup

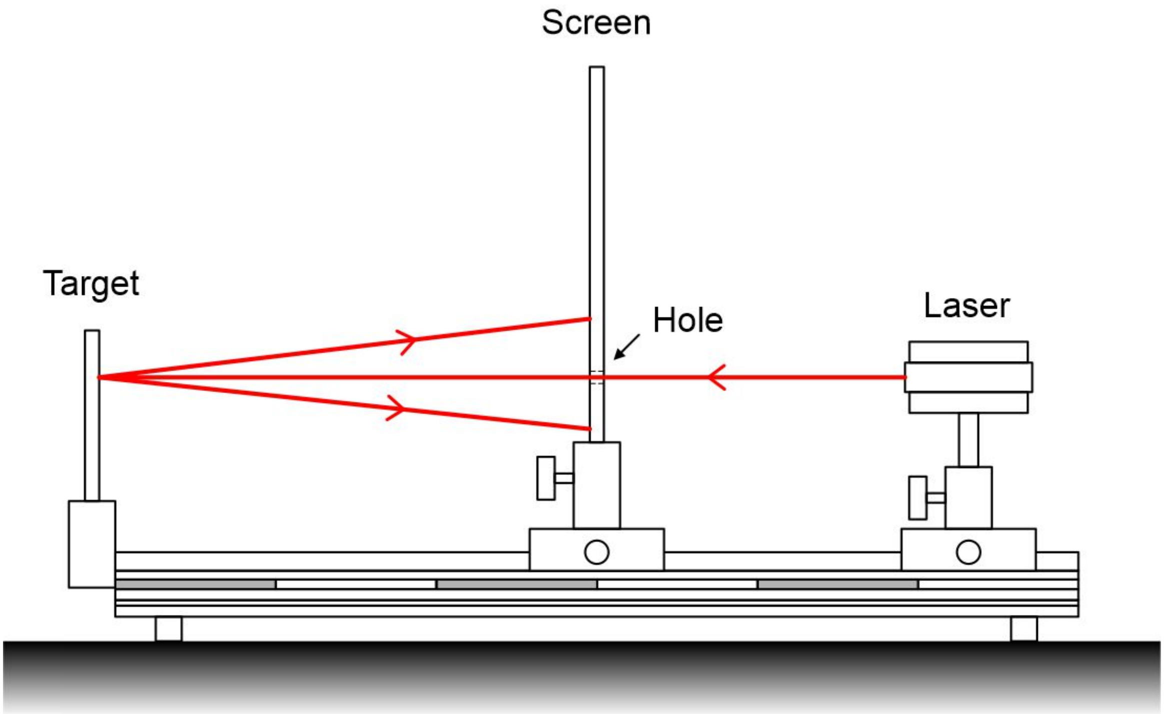


Figure 1a. Lateral view of the setup. The laser beam passes through a hole in a semi-transparent screen and is then directly reflected onto the target, allowing observation of the diffraction pattern of the reflected light.

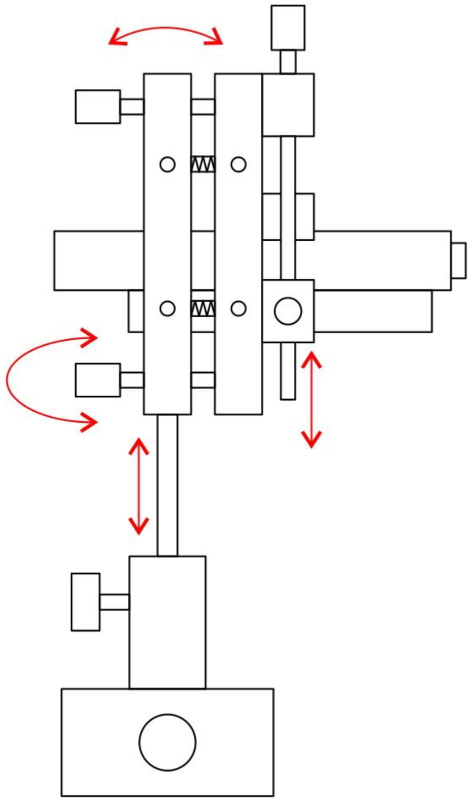


Fig 1b. Sideview of the laser stand. The screws that can be moved along the axis are shown by red arrows.

The experiment consists of two parts. The first part involves studying parameters such as angular diameter and number of fringes, and angular width between the interference fringes as a function of the power supplied to the laser.

The second part focuses on studying the parameters of the aperture (including diameter, height, and shape) that generates the interference pattern, as functions of the power supplied to the laser. Additionally, the parameters of the reference aperture will be determined.

Experimental tools

1. Securely twist the string that was previously used to suspend the test ball onto the top fastener, detach the mechanical part, and place it on a soft mat.

2. Place the laser, screen and acrylic base on the optical bench as shown in the picture. Position the laser beam along the axis of the optical bench at the same level, allowing the beam to pass freely through the hole of the screen. Make sure that the beam is nearly perpendicular to the acrylic surface.

3. Fix the acrylic plate with a magnetic holder. An acrylic white paper surface is used to focus the beam and pre-determine the shooting coordinates.

Note 1: During shooting, gradually increase the power starting from a low setting and make sure to avoid direct exposure of bright light to your eyes.

Note 2: Ensure that the surface is thoroughly clean. Use the provided lens cleaning wipes to clean the shooting area, be careful not to wipe the acrylic surface with a rag, as it can easily become charged through friction and attract dust.

4. Position the flat plate with its opposite surface facing the laser beam, remove the sticker, and accurately determine the shooting point. Please be aware that if the surface is dirty or exposed to excessively bright light, the interference fringes may become distorted and oval, and additional fringes may also form.

5. Move the power switch to "ON" position.

6. By changing the voltage falling on the laser diode, increase the light intensity. Connect the multimeter to the laser circuit to measure DC current and DC voltage. Adjust the measurement range from the highest setting to the lowest. The voltage can be increased up to 45 V.

7. A laser beam is reflected directly on a non-transparent, glossy, flat acrylic surface. As the light intensity is gradually increased, the surface will begin to melt at a certain intensity value. Melted spots can be detected visually.

8. Rotate the screw in front of the laser around the axis to adjust the focus on the surface.

9. The light spot created by the reflected radiation from the surface should be created and observed on a flat screen positioned behind the laser. Observe how the shape and size of the spots change by increasing or decreasing the intensity of the light.

Part A [0.8 points]

The resulting pattern exhibits reversibility and shrinkage up to a certain power value. The upper boundary value corresponding to the thermo-elastic range which is known as yield strength should be determined.

A.1	Determine the power associated with this yield strength (p_{max}).	0.3pt

A.2	Determine the diameter of the outermost bright fringe when the laser power is set to the level associated with the yield strength.	0.5pt

Part B [3.5 points]

B.1	The diameter of the outermost light fringe and the number of fringes formed in this test are measured in relation to the power and the results should be recorded in an Answer sheet table.	1.5pt

B.2	Construct a graph depicting the relationship between the diameter of the outermost light interference fringe on the screen and the corresponding power level.	1.0pt

B.3	Plot the number of interference fringes on the screen is measured as a function of power.	1.0pt

Part C [3.7 points]

C.1	Measure the angular width (an angle between the ray of nth order fringe and the ray of $n+1$th order fringe) and visible angle (an angle between the ray of nth order fringe and x-axis) of the dark fringe at a constant power level, depending on the number of the fringe, and record the results in Answer Sheet Table.	1.2pt

C.2	Plot a linear graph of the relationship between the visible angle vs order of fringe.	1.0pt

C.3	Find the slope and Y-intercept of the graph plotted in Task C.2.	0.5pt

C.4	Construct a graph of angular width as a function of the order of fringes.	1.0pt

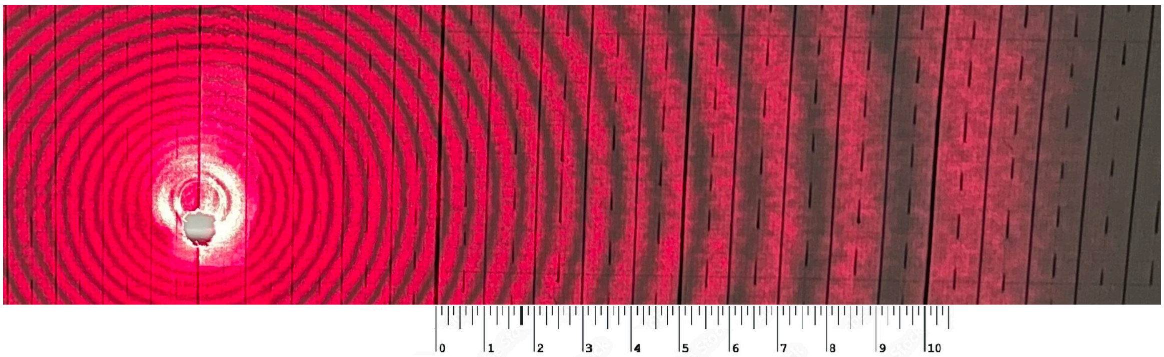
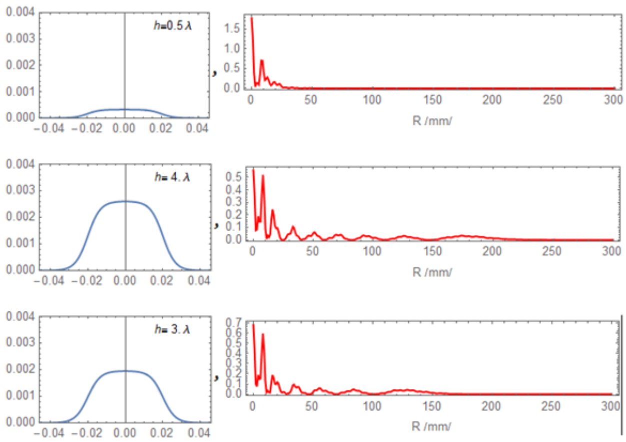


Fig 2. Image of interference fringes on the screen

Part D [2.0 points]

In this section we will use the interference pattern to determine parameters of the thermal deformations. When the laser heats the surface this causes a deformation which creates an interference pattern on the screen as shown in the diagram below:



The graph in blue shows the cross section of the profile of deformation. The graph shown in red shows the intensity from the centre of the interference pattern. As the intensity increases, the height of the deformation increases and the number of bright interference fringes also increases. As we can see in the figure there is an empirical relationship between the height and the number of fringes which is $m = 2h/\lambda$

D.1	By counting the number of the fringes determine the highest order of the fringes. Determine the height of the thermal deformation in terms of the laser wavelength as a function of the laser power. Plot a graph of your data. Hint: ensure your data includes the range of $200mW$ to $400mW$.	1.4pt

D.2	What are the thermal deformation heights for the following input laser powers? Give your answers in units of the number of laser wavelengths. • $200mW$ • $300mW$ • $400mW$	0.6pt