IPhO2009

Experimental Competition
Wednesday, July 15, 2009

The experimental part of this Olympiad consists of two problems. In Problem 1 the aim is to measure the wavelength of a diode laser, and in Problem 2 the goal is to measure the birefringence of a material called mica.

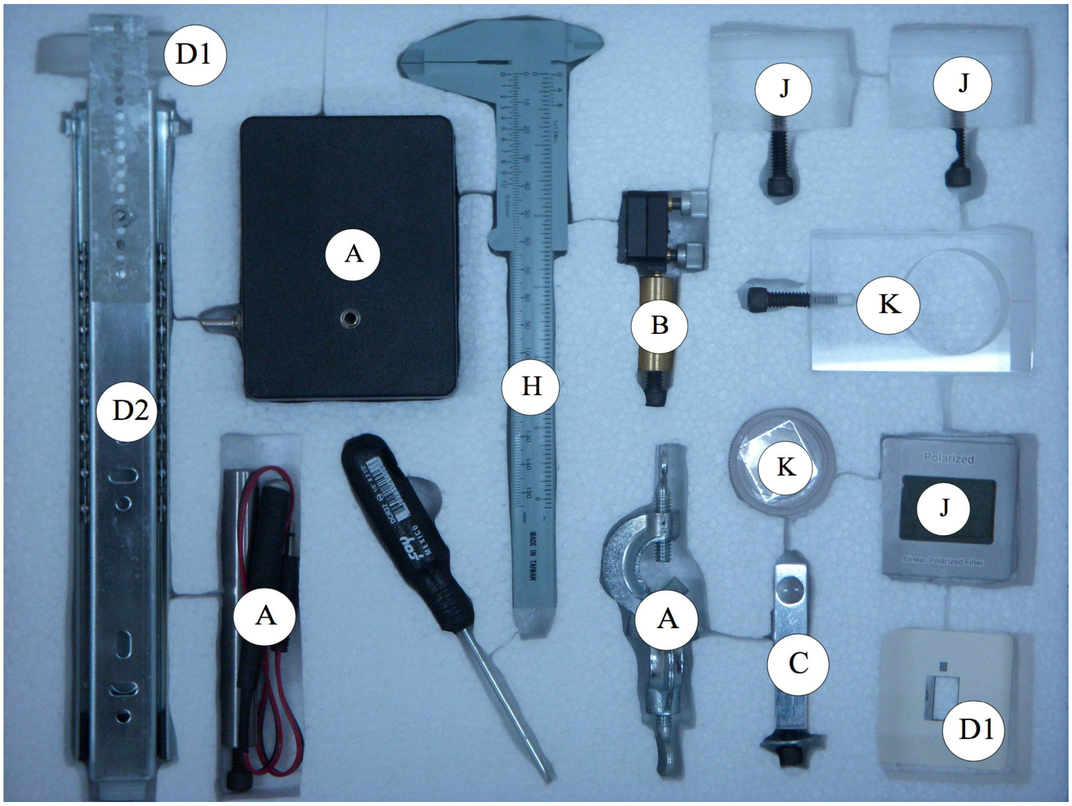
Please read this first:

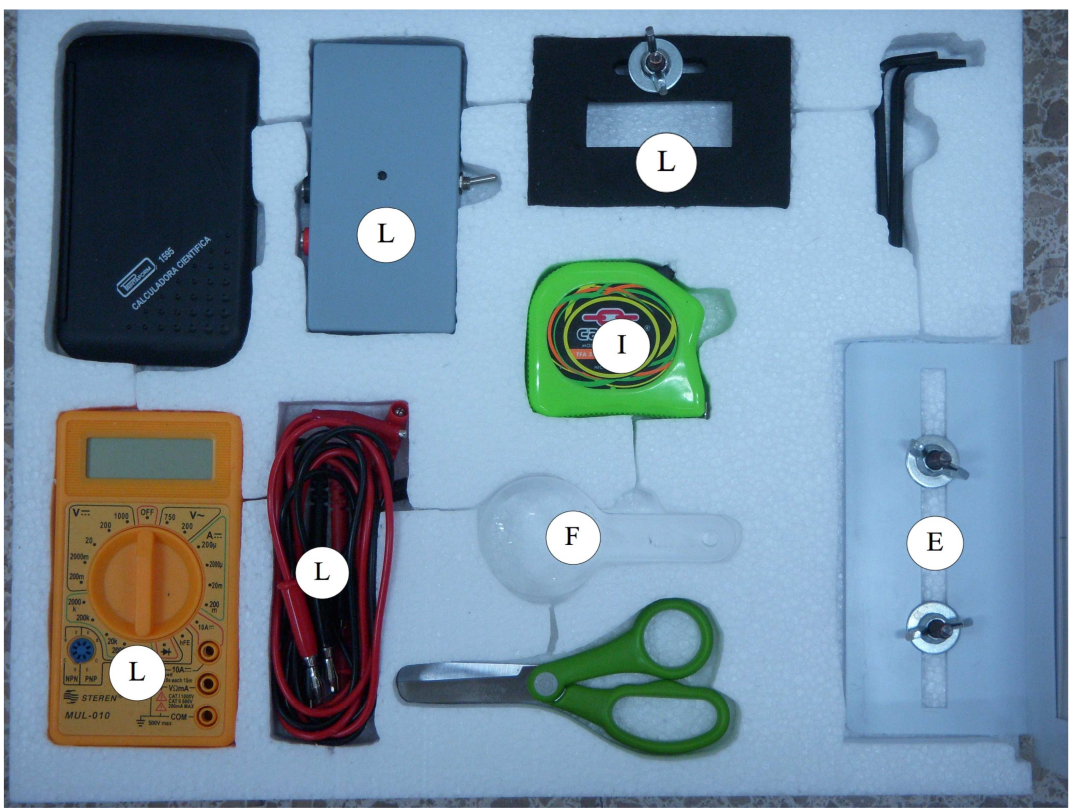
1. The total time available is 5 hours for the experimental competition.
2. Use only the pencils provided.
3. Use only the front side of the paper sheets.
4. Each problem is presented in the ***question form***, marked with a **Q** in the upper left corner.
5. You *must* summarize the answers you have obtained in the **a*nswer form,*** marked with an **A** in the upper left corner.
6. In addition, there is a set of ***working sheets,*** marked with a **W** in the upper left corner, where you may write your calculations.
7. In addition, write down the ***Problem Number*** (1 or 2) on the top of the ***answer forms*** and ***working sheets.***
8. Write on the working sheets of paper whatever you consider is required for the solution of the problem. Please use *as little text as possible*; express yourself primarily in equations, numbers, figures, and plots.
9. For each problem and each of the forms (question form, answer form and working sheets), fill in the boxes at the top of each sheet of paper used by writing your student number (***Student Number***), the progressive page number ***(Page No.)*** and the total number of pages used ***(Total No. of Pages).*** If you use some working sheets of paper for notes that you do not wish to be marked, do **not** destroy it. Instead, mark it with a large X across the entire working sheet and do **not** include it in your numbering.
10. At the end of the exam, arrange all sheets for each problem *in the following order:*

- answer form (including graph paper for your plots).
- used working sheets in order
- the working sheets that you do not wish to be considered (marked with the large X)
- unused working sheets
- printed question form.

Place the papers of each problem set inside the folder and leave everything on your desk. You are not allowed to take *any* sheets of paper out of the room nor any device of the experimental kit.

11. The devices and materials for the experiments are contained in **two** separate packing layers within the box. The photographs of the sets are in the next page. Some devices are LABELED. For each experiment check that all the material is in the box. **If during the experiments you find that any of the devices is not working properly, please ask for a replacement.**


D1
A
H
J
J
B
K
D2
A
K
Polarized
J
Linear Polarized Filter
A
C
D1


L
L
1595
CALCULADORA CIENTIFICA
L
I
F
E
L
STEREN
MUL-010

DIODE LASER EQUIPMENT AND MOVABLE MIRROR.

In both experimental setups you should need a diode laser, with its holder and power supply, and a mirror on a mechanical movable mount.

Before you decide on which problem to work first, we suggest that you mount the laser and the mirror, as indicated in Figure 0. Use the following material:

1) Wooden optical table.
2) Diode laser equipment. Includes the diode laser, support post, "S" clamp and power supply box (LABEL A). See photograph for mounting instructions. **DO NOT LOOK DIRECTLY INTO THE LASER BEAM.**
3) A mirror on a movable mount with two adjusting knobs and support post (LABEL B). See photograph for mounting instructions. **CAUTION: mount the support post to the optical table without touching the mirror. Take off the paper cover after you have mounted the mirror.**

Mount the above devices as indicated in Figure 0. The alignment of the laser beam will be done later on. NOTE: Although we have provided you with optional Allen wrenches, everything can be left fingertight.

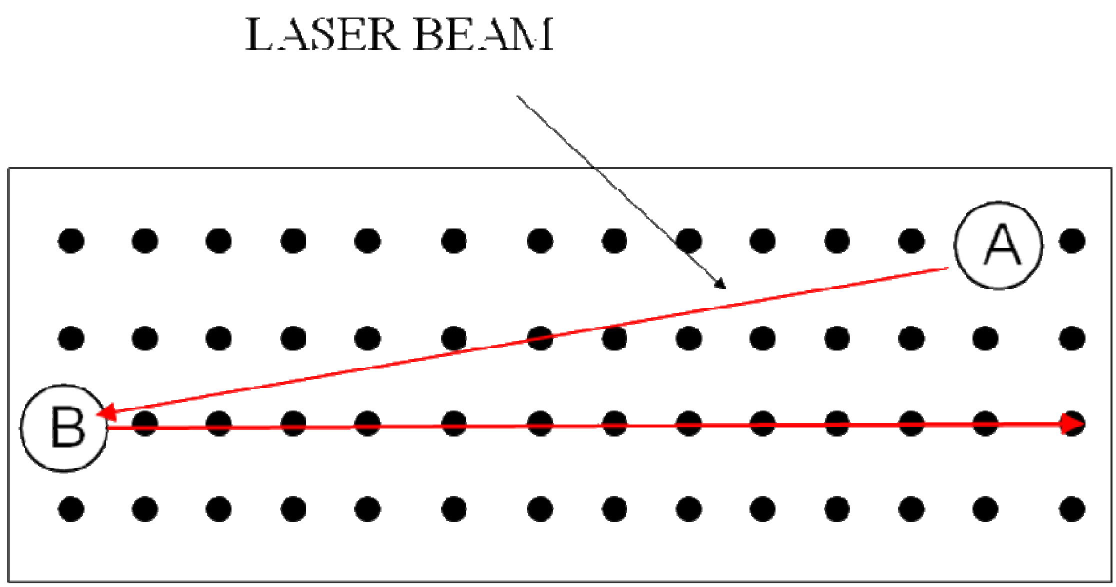
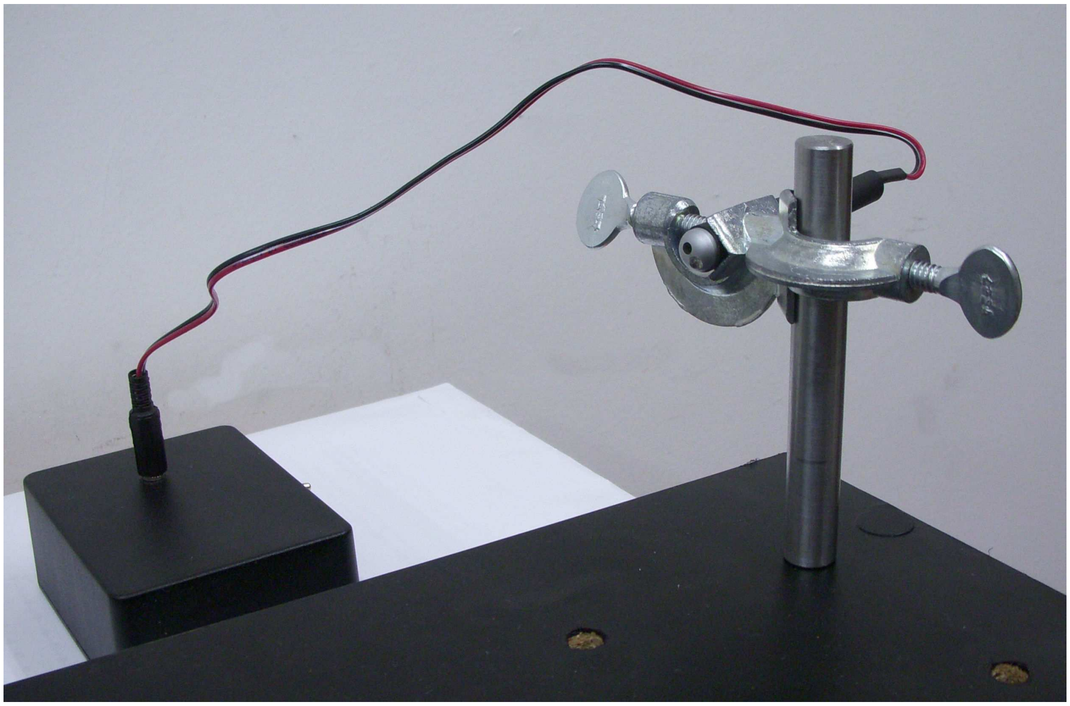
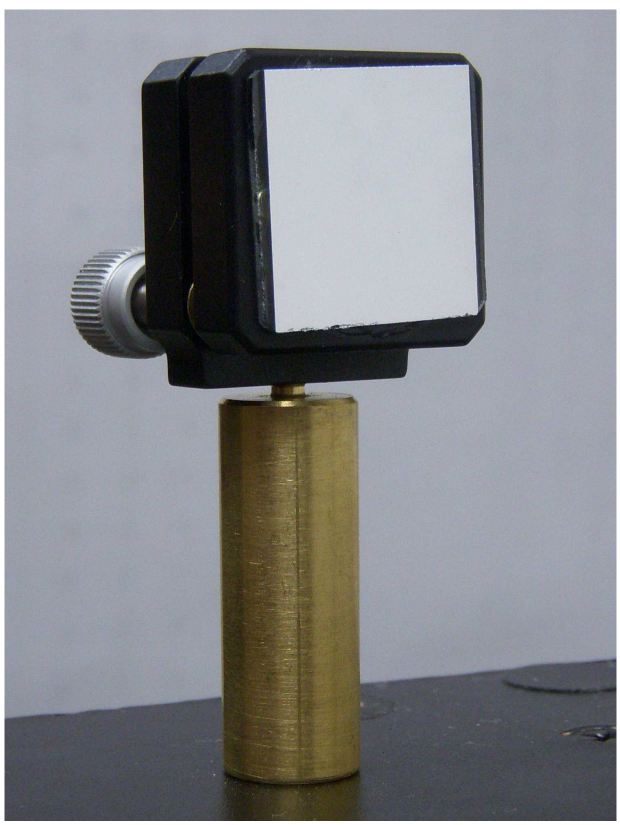


Figure 0. Mounting the laser and the mirror.


Diode laser, support post, "S" clamp and power supply box (LABEL A).


Mirror on a movable movable mount with two adjusting knobs and support post (LABEL B).

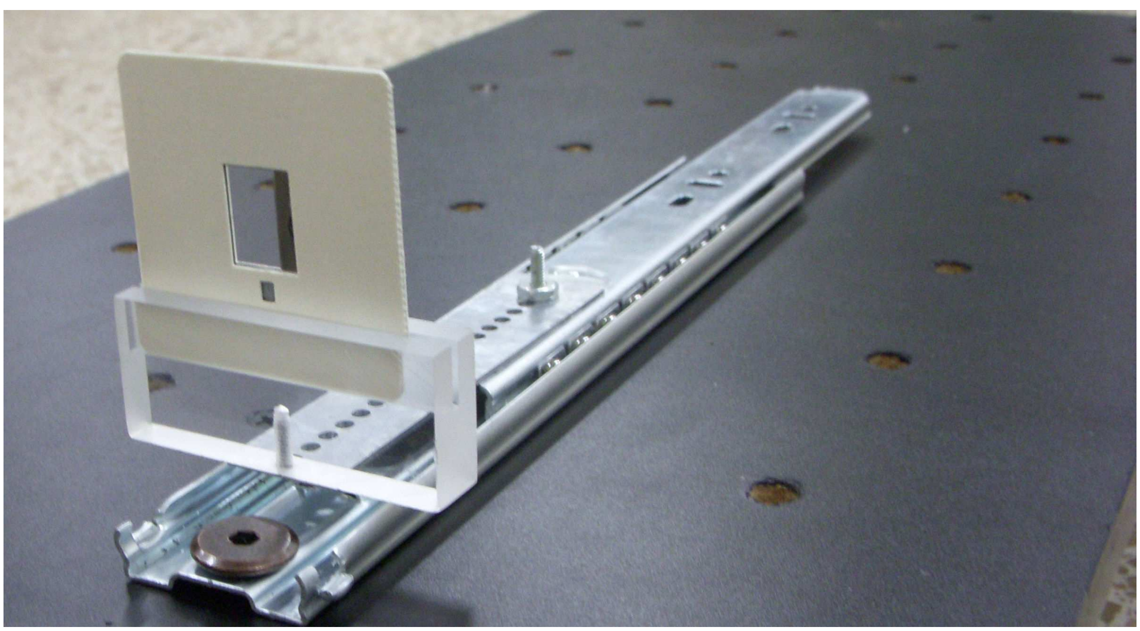
EXPERIMENTAL PROBLEM 1

DETERMINATION OF THE WAVELENGTH OF A DIODE LASER

MATERIAL

In addition to items 1), 2) and 3), you should use:

4) A lens mounted on a square post (LABEL C).
5) A razor blade in a slide holder to be placed in acrylic support, (LABEL D1) and mounted on sliding rail (LABEL D2). Use the screwdriver to tighten the support if necessary. See photograph for mounting instructions.
6) An observation screen with a caliper scale (1/20 mm) (LABEL E).
7) A magnifying glass (LABEL F).
8) 30 cm ruler (LABEL G).
9) Caliper (LABEL H).
10) Measuring tape (LABEL I).
11) Calculator.
12) White index cards, masking tape, stickers, scissors, triangle squares set.
13) Pencils, paper, graph paper.



Razor blade in a slide holder to be placed in acrylic support (LABEL D1) and mounted on sliding rail (LABEL D2).

EXPERIMENT DESCRIPTION

You are asked to determine a diode laser wavelength. The particular feature of this measurement is that no exact micrometer scales (such as prefabricated diffraction gratings) are used. The smallest lengths measured are in the millimetric range. The wavelength is determined using light diffraction on a sharp edge of a razor blade.

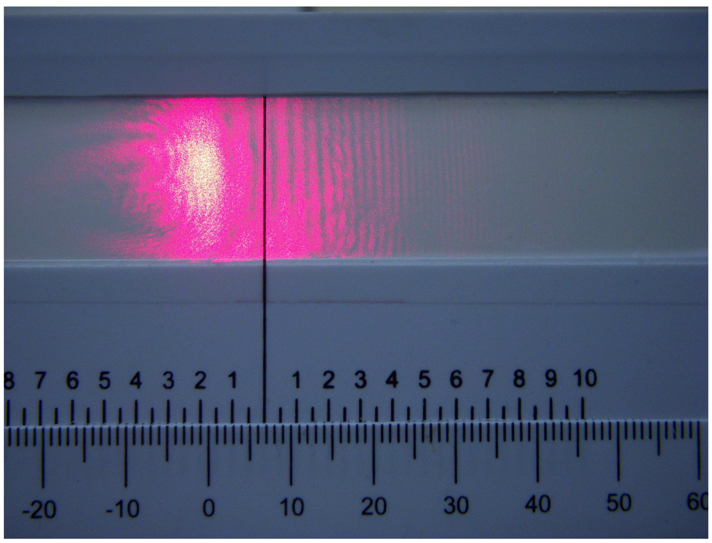


Figure 1.1 Typical interference fringe pattern.

Once the laser beam (A) is reflected on the mirror (B), it must be made to pass through a lens (C), which has a focal length of *a few centimeters*. It can now be assumed that the focus is a light point source from which a spherical wave is emitted. After the lens, and along its path, the laser beam hits a sharp razor blade edge as an obstacle. This can be considered to be a light source from which a cylindrical wave is emitted. These two waves interfere with each other, in the forward direction, creating a diffractive pattern that can be observed on a screen. See Figure 1.1 with a photograph of a typical pattern.

There are two important cases, see Figures 1.2 and 1.3.

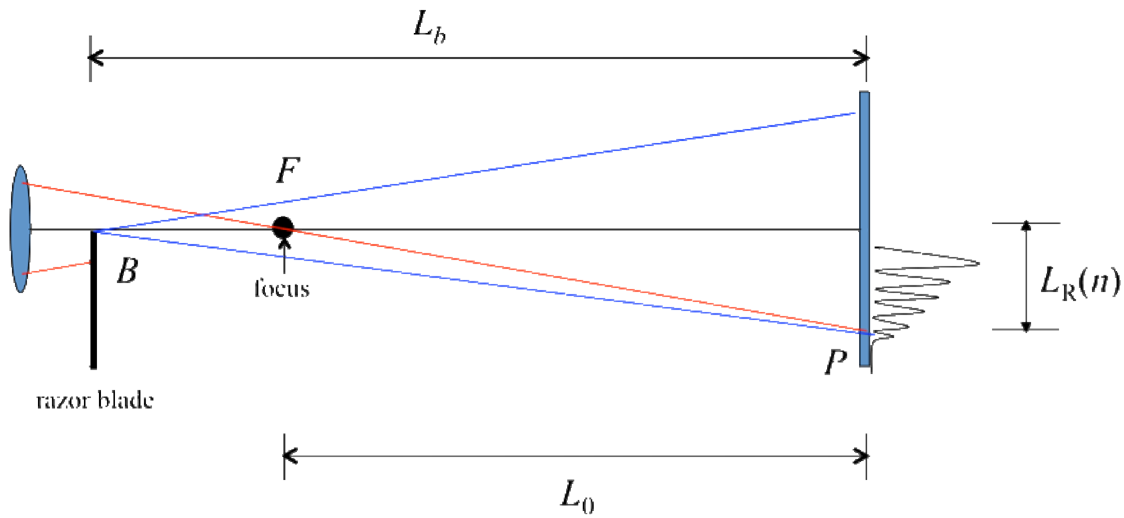


Figure 1.2. Case (I). The razor blade is *before* the focus of the lens. Figure is not at scale. B in this diagram is the edge of the blade and F is the focal point.

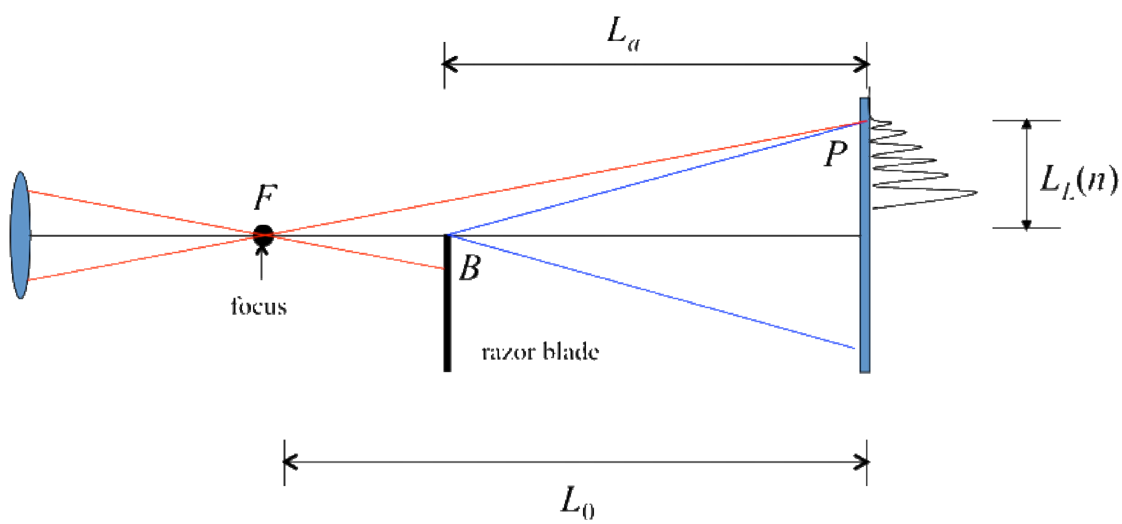


Figure 1.3. Case (II). The razor blade is *after* the focus of the lens. Figure is not at scale. B in this diagram is the edge of the blade and F is the focal point.

EXPERIMENTAL SETUP

Task 1.1 Experimental setup (1.0 points). Design an experimental setup to obtain the above described interference patterns. The distance L_0 from the focus to the screen should be much larger than the focal length.

- Make a sketch of your experimental setup in the drawing of the optical table provided. Do this by writing the LABELS of the different devices on the drawing of the optical table. You can make additional simple drawings to help clarify your design.
- You may align the laser beam by using one of the white index cards to follow the path.
- Make a sketch of the laser beam path on the drawing of the optical table and write down the height h of the beam as measured from the optical table.

WARNING: Ignore the larger circular pattern that may appear. This is an effect due to the laser diode itself.

Spend some time familiarizing yourself with the setup. You should be able to see of the order of 10 or more vertical linear fringes on the screen. The readings are made using the positions of the **dark** fringes. You may use the magnifying glass to see more clearly the position of the fringes. **The best way to observe the fringes is to look at the back side of the illuminated screen (E).** Thus, the scale of the screen should face out of the optical table. If the alignment of the optical devices is correct, you should see both patterns (of Cases I and II) by simply sliding the blade (D1) through the rail (D2).

THEORETICAL CONSIDERATIONS

Refer to Figure 1.2 and 1.3 above. There are five basic lengths:

L_0 : distance from the focus to the screen.
L_b : distance from the razor blade to the screen, Case I.
L_a : distance from the razor blade to the screen, Case II.
$L_R(n)$: position of the *n*-th **dark** fringe for Case I.
$L_L(n)$: position of the *n*-th **dark** fringe for Case II.

The first dark fringe, for both Cases I and II, is the widest one and corresponds to $n = 0$.

Your experimental setup must be such that $L_R(n) << L_0, L_b$ for Case I and $L_L(n) << L_0, L_a$ for Case II.

The phenomenon of wave interference is due to the difference in optical paths of a wave starting at the same point. Depending on their phase difference, the waves may cancel each other (destructive interference) giving rise to dark fringes; or the waves may add (constructive interference) yielding bright fringes.

A detailed analysis of the interference of these waves gives rise to the following condition to obtain a **dark** fringe, for Case I:

$$\Delta_{\mathrm{I}}(n) = \left(n + \frac{5}{8}\right)\lambda \qquad \text{with} \qquad n = 0, 1, 2, \ldots \qquad (1.1)$$

and for Case II:

$$\Delta_{\mathrm{II}}(n) = \left(n + \frac{7}{8}\right)\lambda \qquad \text{with} \qquad n = 0, 1, 2, \ldots \qquad (1.2)$$

where λ is the wavelength of the laser beam, and Δ_{I} and Δ_{II} are the optical path differences for each case.

The difference in optical paths for Case I is,

$$\Delta_{\mathrm{I}}(n) = (BF + FP) - BP \qquad \text{for each} \qquad n = 0, 1, 2, \ldots \qquad (1.3)$$

while for Case II is,

$$\Delta_{\mathrm{II}}(n) = (FB + BP) - FP \qquad \text{for each} \qquad n = 0, 1, 2, \ldots \qquad (1.4)$$

Task 1.2 Expressions for optical paths differences (0.5 points). Assuming $L_R(n) << L_0, L_b$ for Case I and $L_L(n) << L_0, L_a$ for Case II in equations (1.3) and (1.4) (make sure your setup satisfies these conditions), find approximated expressions for $\Delta_{\mathrm{I}}(n)$ and $\Delta_{\mathrm{II}}(n)$ in terms of L_0, L_b, L_a, $L_R(n)$ and $L_L(n)$. You may find useful the approximation $(1+x)^r \approx 1 + rx$ if $x << 1$.

The experimental difficulty with the above equations is that L_0, $L_R(n)$ and $L_L(n)$ cannot be accurately measured. The first one because it is not easy to find the position of the focus of the lens, and the two last ones because the origin from which they are defined may be very hard to find due to misalignments of your optical devices.

To solve the difficulties with $L_R(n)$ and $L_L(n)$, first choose the zero (0) of the scale of the screen (LABEL E) as the origin for all your measurements of the fringes. Let l_{0R} and l_{0L} be the (unknown) positions from which $L_R(n)$ and $L_L(n)$ are defined. Let $l_R(n)$ and $l_L(n)$ be the positions of the fringes as measured from the origin (0) you chose. Therefore

$$L_R(n) = l_R(n) - l_{0R} \qquad \text{and} \qquad L_L(n) = l_L(n) - l_{0L} \qquad (1.5)$$

PERFORMING THE EXPERIMENT. DATA ANALYSIS.

Task 1.3 Measuring the dark fringe positions and locations of the blade (3.25 points).

- For both Case I and Case II, measure the positions of the dark fringes $l_R(n)$ and $l_L(n)$ as a function of the number fringe *n*. Write down your measurements in Table I; you should report no less than 8 measurements for each case.
- Report also the positions of the blade L_b and L_a, and indicate with its LABEL the intrument you used.
- **IMPORTANT SUGGESTION:** For purposes of both simplification of analysis and better accuracy, measure *directly* the distance $d = L_b - L_a$ with a better accuracy than that of L_b and L_a; that is, do not calculate it from the measurements of L_b and L_a. Indicate with its LABEL the instrument you used.

Make sure that you include the uncertainty of your measurements.

Task 1.4 Data analysis. (3.25 points). With all the previous information you should be able to find out the values of l_{0R} and l_{0L}, and, of course, of the wavelength λ.

- Devise a procedure to obtain those values. Write down the expressions and/or equations needed.
- Include the analysis of the errors. You may use Table I or you can use another one to report your findings; make sure that you label clearly the contents of the columns of your tables.
- Plot the variables analyzed. Use the graph paper provided.
- Write down the values for l_{0R} and l_{0L}, with uncertainties.

Task 1.5 Calculating λ. Write down the calculated value for λ. Include its uncertainty and the analysis to obtain it. **SUGGESTION:** In your formula for λ, wherever you find $(L_b - L_a)$ replace it by d and use its measured value. **(2 points).**

EXPERIMENTAL PROBLEM 2
BIREFRINGENCE OF MICA

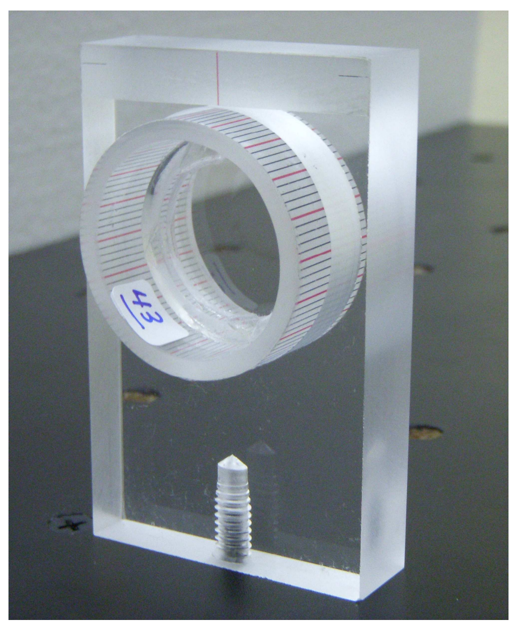
In this experiment you will measure the birefringence of mica (a crystal widely used in polarizing optical components).

MATERIAL

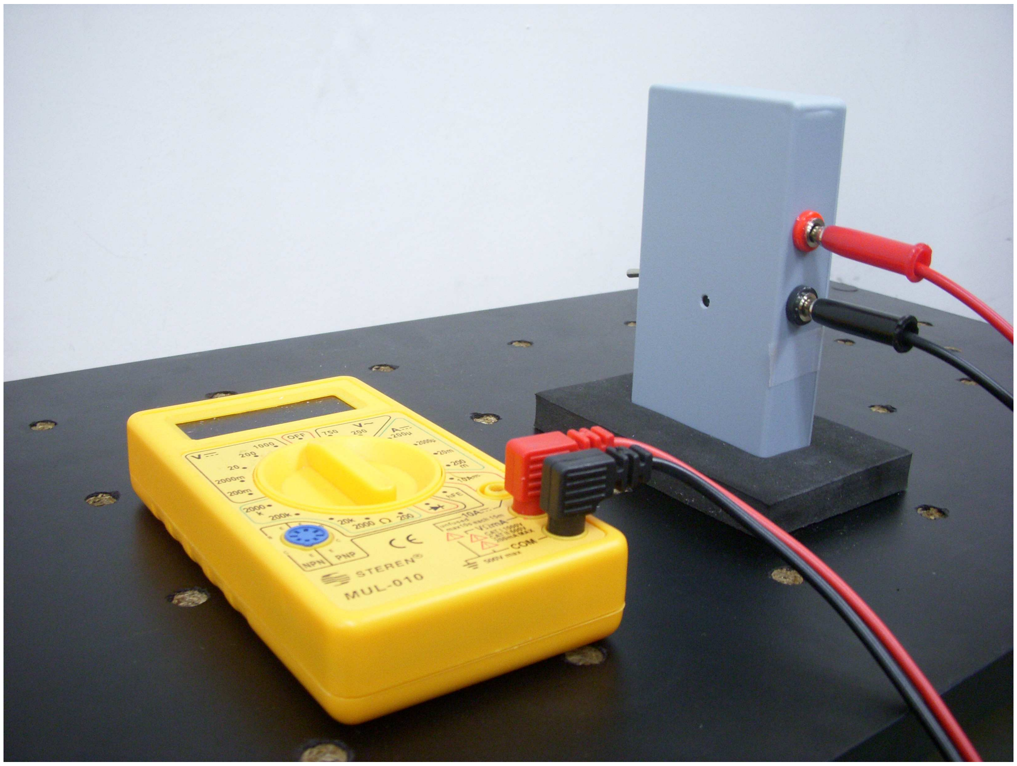
In addition to items 1), 2) and 3), you should use,

14) Two polarizing films mounted in slide holders, each with an additional acrylic support (LABEL J). See photograph for mounting instructions.
15) A thin mica plate mounted in a plastic cylinder with a scale with no numbers; acrylic support for the cylinder (LABEL K). See photograph for mounting instructions.
16) Photodetector equipment. A photodetector in a plastic box, connectors and foam support. A multimeter to measure the voltage of the photodetector (LABEL L). See photograph for mounting and connecting instructions.
17) Calculator.
18) White index cards, masking tape, stickers, scissors, triangle squares set.
19) Pencils, paper, graph paper.

Polarizer mounted in slide holder with acrylic support (LABEL J).



Thin mica plate mounted in cylinder with a scale with no numbers, and acrylic support (LABEL K).



A photodetector in a plastic box, connectors and foam support. A multimeter to measure the voltage of the photodetector (LABEL L). Set the connections as indicated.

DESCRIPTION OF THE PHENOMENON

Light is a transverse electromagnetic wave, with its electric field lying on a plane perpendicular to the propagation direction and oscillating in time as the light wave travels.

If the direction of the electric field remains in time oscillating *along a single line*, the wave is said to be linearly polarized, or simply, polarized. See Figure 2.1.

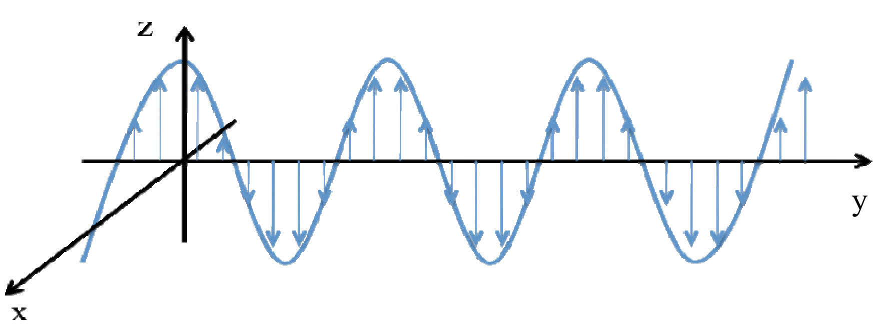


Figure 2.1 A wave travelling in the y-direction and polarized in the z-direction.

A polarizing film (or simply, a polarizer) is a material with a privileged axis parallel to its surface, such that, transmitted light emerges polarized along the axis of the polarizer. Call (+) the privileged axis and (-) the perpendicular one.

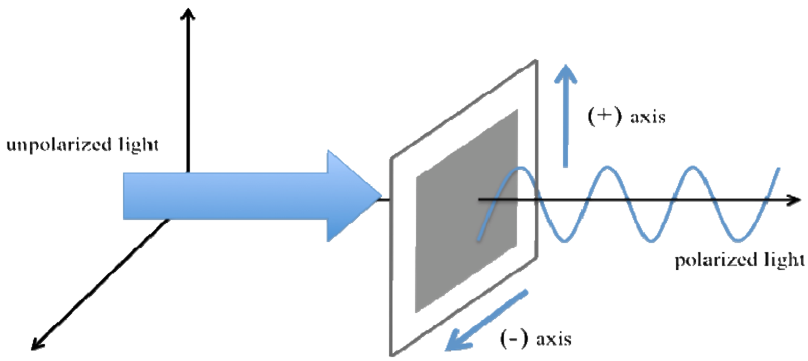


Figure 2.2 Unpolarized light normally incident on a polarizer. Transmitted light is polarized in the (+) direction of the polarizer.

Common transparent materials (such as window glass), transmit light with the same polarization as the incident one, because its index of refraction does not depend on the direction and/or polarization of the incident wave. Many crystals, including mica, however, are sensitive to the direction of the electric field of the wave. For propagation perpendicular to its surface, the mica sheet has two characteristic orthogonal axes, which we will call Axis 1 and Axis 2. This leads to the phenomenon called birefringence.

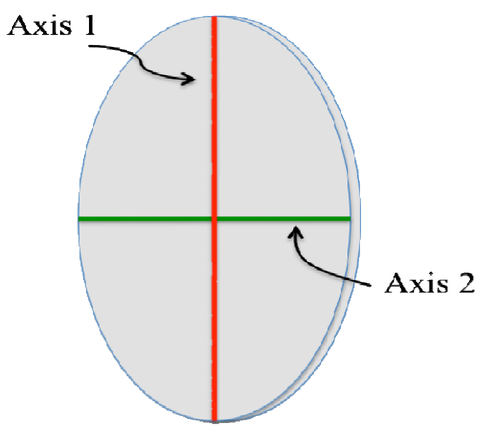


Figure 2.3 Thin slab of mica with its two axes, Axis 1 (red) and Axis 2 (green).

Let us analyze two simple cases to exemplify the birefringence. Assume that a wave **polarized in the vertical direction** is normally incident on one of the surfaces of the thin slab of mica.

Case 1) Axis 1 or Axis 2 is parallel to the polarization of the incident wave. The trasmitted wave passes without changing its polarization state, but the propagation is characterized as if the material had either an index of refraction n_1 or n_2. See Figs. 2.4 and 2.5.

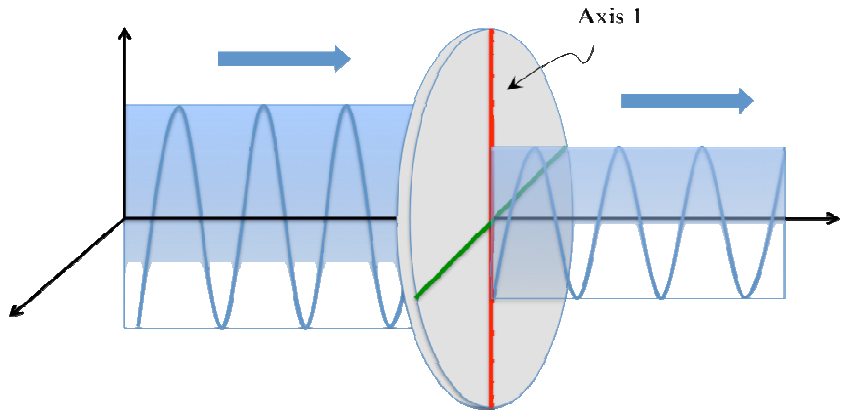


Figure 2.4 Axis 1 is parallel to polarization of incident wave. Index of refraction is n_1.

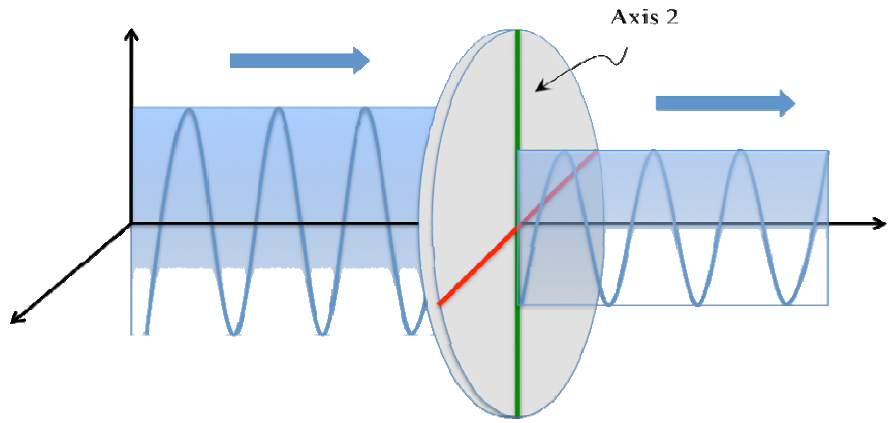


Figure 2.5 Axis 2 is parallel to polarization of incident wave. Index of refraction is n_2.

Case 2) Axis 1 makes an angle θ with the direction of polarization of the incident wave. The transmitted light has a more complicated polarization state. This wave, however, can be seen as the *superposition* of two waves with different phases, one that has polarization **parallel** to the polarization of the incident wave (i.e. "vertical") and another that has polarization **perpendicular** to the polarization of the incident wave (i.e. "horizontal").

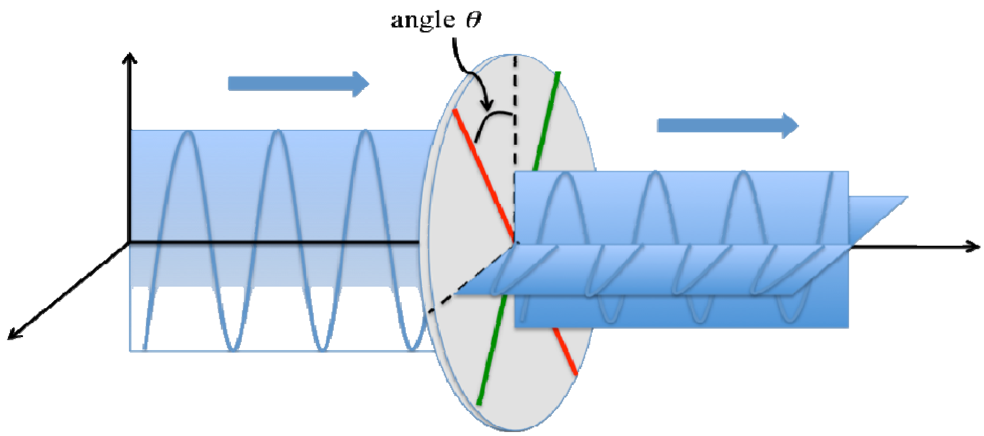


Figure 2.6 Axis 1 makes and angle θ with polarization of incident wave

Call I_P the *intensity* of the wave transmitted *parallel* to the polarization of the incident wave, and I_O the *intensity* of the wave transmitted *perpendicular* to polarization of the incident wave. These intensities depend on the angle θ, on the wavelength λ of the light source, on the thickness L of the thin plate, and on the absolute value of the difference of the refractive indices, $|n_1 - n_2|$. This last quantity is called the *birefringence* of the material. The measurement of this quantity is the goal of this problem. Together with polarizers, birefringent materials are useful for the control of light polarization states.

We point out here that the photodetector measures the intensity of the light incident on it, independent of its polarization.

The dependence of $I_P(\theta)$ and $I_O(\theta)$ on the angle θ is complicated due to other effects not considered, such as the absorption of the incident radiation by the mica. One can obtain, however, approximated but very simple expressions for the normalized intensities $\bar{I}_P(\theta)$ and $\bar{I}_O(\theta)$, defined as,

$$\bar{I}_P(\theta) = \frac{I_P(\theta)}{I_P(\theta) + I_O(\theta)} \qquad (2.1)$$

and

$$\bar{I}_O(\theta) = \frac{I_O(\theta)}{I_P(\theta) + I_O(\theta)} \qquad (2.2)$$

It can be shown that the normalized intensities are (approximately) given by,

$$\bar{I}_P(\theta) = 1 - \frac{1}{2}(1 - \cos\Delta\phi)\sin^2(2\theta) \qquad (2.3)$$

and

$$\bar{I}_O(\theta) = \frac{1}{2}(1 - \cos\Delta\phi)\sin^2(2\theta) \tag{2.4}$$

where $\Delta\phi$ is the difference of phases of the parallel and perpendicular transmitted waves. This quantity is given by,

$$\Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2| \tag{2.5}$$

where L is the thickness of the thin plate of mica, λ the wavelength of the incident radiation and $|n_1 - n_2|$ the birefringence.

EXPERIMENTAL SETUP

Task 2.1 Experimental setup for measuring intensities. Design an experimental setup for measuring the intensities I_P and I_O of the transmitted wave, as a function of the angle θ of any of the optical axes, as shown in Fig. 2.6. *Do this by writing the LABELS of the different devices on the drawing of the optical table.* Use the convention (+) and (-) for the direction of the polarizers. You can make additional simple drawings to help clarify your design.

Task 2.1 a) Setup for I_P **(0.5 points).**
Task 2.1 b) Setup for I_O **(0.5 points).**

Laser beam alignment. Align the laser beam in such a way that it is parallel to the table and is incident on the center of the cylinder holding the mica. You may align by using one the white index cards to follow the path. Small adjustments can be made with the movable mirror.

Photodetector and the multimeter. The photodetector produces a voltage as light impinges on it. Measure this voltage with the multimeter provided. The voltage produced is linearly proportional to the intensity of the light. Thus, report the intensities as the voltage produced by the photodetector. Without any laser beam incident on the photodetector, you can measure the background light intensity of the detector. This should be less than 1 mV. *Do not correct* for this background when you perform the intensity measurements.

WARNING: The laser beam is partially polarized but it is not known in which direction. Thus, to obtain polarized light with good intensity readings, place a polarizer with either its (+) or (–) axes vertically in such a way that you obtain the maximum transmitted intensity in the absence of any other optical device.

MEASURING INTENSITIES

Task 2.2 The scale for angle settings. The cylinder holding the mica has a regular graduation for settings of the angles. Write down the value in degrees of the smallest interval (*i.e.* between two black consecutive lines). **(0.25 points).**

Finding (approximately) the zero of θ and/or the location of the mica axes. To facilitate the analysis, it is very important that you find the appropriate zero of the angles. We suggest that, first, you identify the location of one of the mica axes, and call it Axis 1. It is almost sure that this position will not coincide with a graduation line on the cylinder. Thus, consider the nearest graduation line in the mica cylinder as the provisional origin for the angles. Call $\overline{\theta}$ the angles measured from such an origin. Below you will be asked to provide a more accurate location of the zero of θ.

Task 2.3 Measuring I_P and I_O. Measure the intensities I_P and I_O for as many angles $\overline{\theta}$ as you consider necessary. Report your measurements in Table I. Try to make the measurements for I_P and I_O for the *same* setting of the cylinder with the mica, that is, for a *fixed* angle $\overline{\theta}$. **(3.0 points).**

Task 2.4 Finding an appropriate zero for θ. The location of Axis 1 defines the zero of the angle θ. As mentioned above, it is mostly sure that the location of Axis 1 does not coincide with a graduation line on the mica cylinder. To find the zero of the angles, you may proceed either graphically or numerically. Recognize that the relationship near a maximum or a minimum may be approximated by a parabola where:

$$I(\overline{\theta}) \approx a\overline{\theta}^2 + b\overline{\theta} + c$$

and the minimum or maximum of the parabola is given by,

$$\overline{\theta}_m = -\frac{b}{2a}.$$

Either of the above choices gives rise to a shift $\delta\overline{\theta}$ of all your values of $\overline{\theta}$ given in Table I of Task 2.3, such that they can now be written as angles θ from the appropriate zero, $\theta = \overline{\theta} + \delta\overline{\theta}$. Write down the value of the shift $\delta\overline{\theta}$ in degrees. **(1.0 points).**

DATA ANALYSIS.

Task 2.5 Choosing the appropriate variables. Choose $\overline{I}_P(\theta)$ or $\overline{I}_O(\theta)$ to make an analysis to find the difference of phases $\Delta\phi$. Identify the variables that you will use. (**0.5 point).**

Task 2.6 Data analysis and the phase difference.

- Use Table II to write down the values of the variables needed for their analysis. Make sure that you use the corrected values for the angles θ. Include uncertainties. Use graph paper to plot your variables. **(1.0 points).**

- Perform an analysis of the data needed to obtain the phase difference $\Delta\phi$. Report your results including uncertainties. Write down any equations or formulas used in the analysis. Plot your results. **(1.75 points).**

- Calculate the value of the phase difference $\Delta\phi$ in radians, including its uncertainty. Find the value of the phase difference in the interval $[0,\pi]$. **(0.5 points).**

Task 2.7 Calculating the birefringence $|n_1 - n_2|$. You may note that if you add $2N\pi$ to the phase difference $\Delta\phi$, with *N* any integer, or if you change the sign of the phase, the values of the intensities are unchanged. However, the value of the birefringence $|n_1 - n_2|$ would change. Thus, to use the value $\Delta\phi$ found in Task 2.6 to correctly calculate the birefringence, you must consider the following:

$$\Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2| \qquad \text{if} \qquad L < 82\times10^{-6}\text{ m}$$

or

$$2\pi - \Delta\phi = \frac{2\pi L}{\lambda}|n_1 - n_2| \qquad \text{if} \qquad L > 82\times10^{-6}\text{ m}$$

where the value L of the thickness of the slab of mica you used is written on the cylinder holding it. This number is given in micrometers (1 micrometer = 10^{-6} m). Assign 1×10^{-6}m as the uncertainty for L. For the laser wavelength, you may use the value you found in Problem 1 or the average value between 620×10^{-9} m and 750×10^{-9} m, the reported range for red in the visible spectrum. Write down the values of L and λ as well as the birefringence $|n_1 - n_2|$ with its uncertainty. Include the formulas that you used to calculate the uncertainties. **(1.0 points).**