Experimental problem 1

There are two experimental problems. The setup on your table is used for both problems. You have 5 hours to complete the entire task (1&2).

Experimental problem 1: Elasticity of sheets

Introduction

Springs are objects made from elastic materials which can be used to store mechanical energy. The most famous helical springs are well described in terms of Hooke's law, which states that the force with which the spring pushes back is linearly proportional to the distance from its equilibrium length: $F = -k\Delta x$, where k is the spring constant, Δx is the displacement from equilibrium, and F is the force [see Fig. 1(a)]. However, elastic springs can have quite different shapes from the usual helical springs, and for larger deformations Hooke's law does not generally apply. In this problem we measure the properties of a spring made from a sheet of elastic material, which is schematically illustrated in Fig. 1(b).

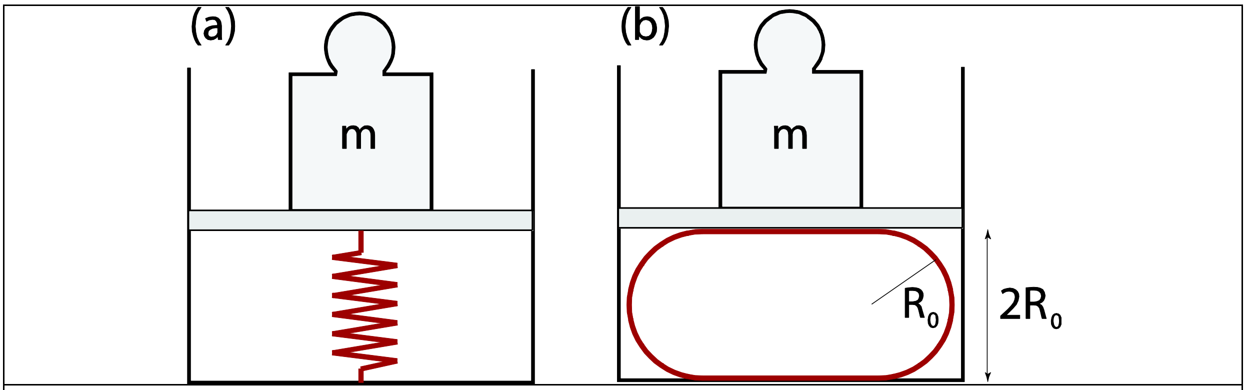

Figure 1. Illustrations of (a) a helical spring and (b) a spring made from a sheet of elastic material rolled up into a cylinder. When the latter spring is sufficiently compressed, its shape can be approximated as a stadium with two semicircles of radii R_0 (see text).

Transparent foil rolled into a cylindrical spring

Suppose that we take a sheet of elastic material (e.g. a transparent foil) and bend it. The more we bend it, the more elastic energy is stored in the sheet. The elastic energy depends on the curvature of the sheet. Parts of the sheet with larger curvature store more energy (flat parts of the sheet do not store energy because their curvature is zero). The springs used in this experiment are made from rectangular transparent foils rolled into cylinders (see Fig. 2). The elastic energy stored in a cylinder is

$E_{el} = \frac{\kappa}{2}\frac{1}{R_c^2}A,$	(1)

where A denotes the area of the cylinder's side (excluding its bases), R_c denotes its radius, and the parameter κ, referred to as the bending rigidity, is determined by the elastic properties of the material and the thickness of the sheet. Here we neglect the stretching of the sheets.

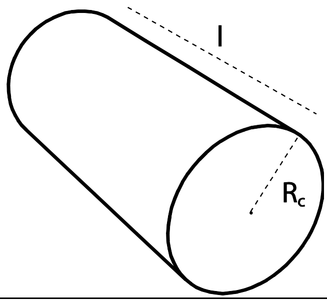

Figure 2. A schematic picture of an elastic sheet rolled into a cylinder of radius R_c and length l.

Suppose that such a cylinder is compressed as in Fig. 1(b). For a given force applied by the press (F), the displacement from equilibrium depends on the elasticity of the transparent foil. For some interval of compression forces, the shape of the compressed transparency foil can be approximated with the shape of a stadium, which has a cross section with two straight lines and two semicircles, both of radius R_0. It can be shown that the energy of the compressed system is minimal when

$$R_0^2 = \frac{l\kappa\pi}{2F}. \qquad (2)$$

The force is measured by the scale calibrated to measure mass m, so $F = mg$, g = 9.81 m/s^2.

Experimental setup (1st problem)

The following items (to be used for the 1st problem) are on your desk:

1. Press (together with a stone block); see separate instructions if needed
2. Scale (measures mass up to 5000 g, it has TARA function, see separate instructions if needed)
3. Transparency foils (all foils are 21 cm x 29.7 cm, the blue foil is 200 µm thick, and the colorless foil is 150 µm thick); please, do ask for the extra foils if you need them.
4. Adhesive (scotch) tape
5. Scissors
6. Ruler with a scale
7. A rectangular wooden plate (the plate is to be placed on a scale, and the foil sits on the plate)

The setup is to be used as in Fig. 3. The upper plate of the press can be moved downward and upward using a wing nut, and the force (mass) applied by the press is measured with the scale. **Important: The wing nut moves 2 mm when rotated 360 degrees.** (Small aluminum rod is not used in Experiment 1.)

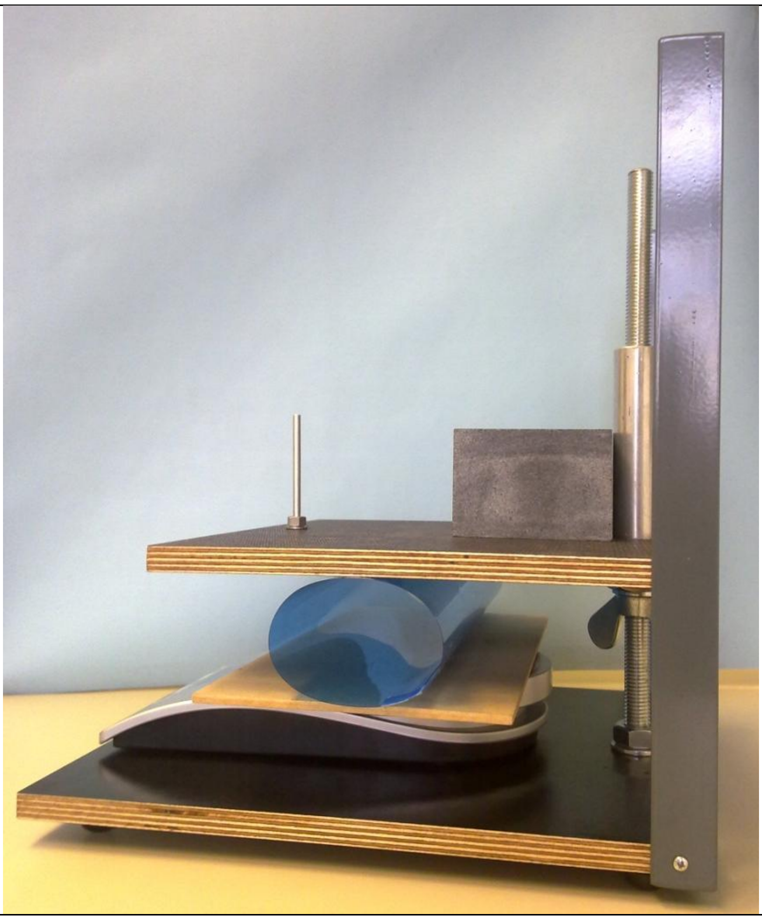

Figure 3. The photo of a setup for measuring the bending rigidity.

Tasks

1. Roll the blue foils into cylinders, one along the longer side, and the other along its shorter side; use the adhesive tape to fix them. The overlap of the sheet should be about 0.5 cm.
   (a) Measure the dependence of the mass read by the scale on the separation between the plates of the press for each of the two cylinders. *(1.9 points)*
   (b) Plot your measurements on appropriate graphs. Using the ruler and eye as the guide, draw lines through the points and determine the bending rigidities κ for the cylinders. Mark the region where the approximate relation (the stadium approximation) holds. Estimate the value of $\frac{R_0}{R_c}$ below which the stadium approximation holds; here R_c is the radius of the non-laden cylinder(s). *(4.3 points)*

   The error analysis of the results is not required.

2. Measure the bending rigidity of a single colorless transparent foil. *(2.8 points)*
3. The bending rigidity κ depends on the Young's modulus Y of elasticity of the isotropic material, and the thickness d of the transparent foil according to

$\kappa = \dfrac{Yd^3}{12(1-\nu^2)}$,	(3)

where ν is the Poisson ratio for the material; for most materials $\nu \approx 1/3$. From the previous measurements, determine the Young's modulus of the blue and the colorless transparent foil. *(1.0 points)*

Experimental problem 2

There are two experimental problems. The setup on your table is used for both problems. You have 5 hours to complete the entire task (1&2).

Experimental problem 2: Forces between magnets, concepts of stability and symmetry

Introduction

Electric current I circulating in a loop of area S creates a magnetic moment of magnitude $m = IS$ [see Fig. 1(a)]. A permanent magnet can be thought of as a collection of small magnetic moments of iron (Fe), each of which is analogous to the magnetic moment of a current loop. This (Ampère's) model of a magnet is illustrated in Fig. 1(b). The total magnetic moment is a sum of all small magnetic moments, and it points from the south to the northern pole.

(a) (b)

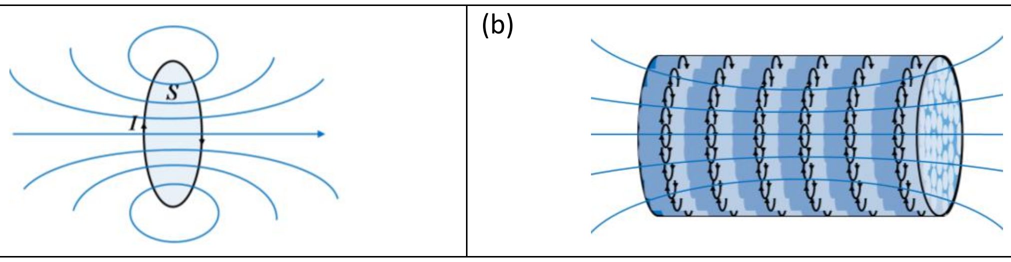

Figure 1. (a) Illustration of a current loop and the produced magnetic field. (b) Ampère's model of a magnet as a collection of small current loops.

Forces between magnets

To calculate the force between two magnets is a nontrivial theoretical task. It is known that like poles of two magnets repel, and unlike poles attract. The force between two current loops depends on the strengths of the currents in them, their shape, and their mutual distance. If we reverse the current in one of the loops, the force between them will be of the same magnitude, but of the opposite direction.

In this problem you experimentally investigate the forces between two magnets, the ring-magnet and the rod-magnet. We are interested in the geometry where the axes of symmetry of the two magnets coincide (see Fig. 2). The rod-magnet can move along the z - axis from the left, through the ring-magnet, and then towards the right (see Fig. 2). Among other tasks, you will be asked to measure the force between the magnets as a function of z. The origin $z = 0$ corresponds to the case when the centers of the magnets coincide.

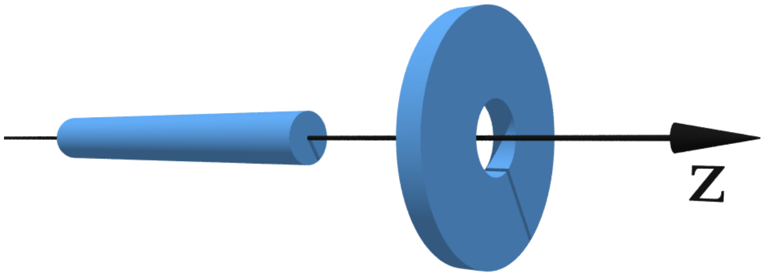

Figure 2. The rod- and the ring-magnet are aligned. The force between them changes as the rod-magnet moves along the z - axis.

To ensure motion of the rod-magnet along the axis of symmetry (z - axis), the ring-magnet is firmly embedded in a transparent cylinder, which has a narrow hole drilled along the z - axis. The rod-magnet is thus constrained to move along the z - axis through the hole (see Fig. 3). The magnetization of the magnets is along the z - axis. The hole ensures radial stability of the magnets.

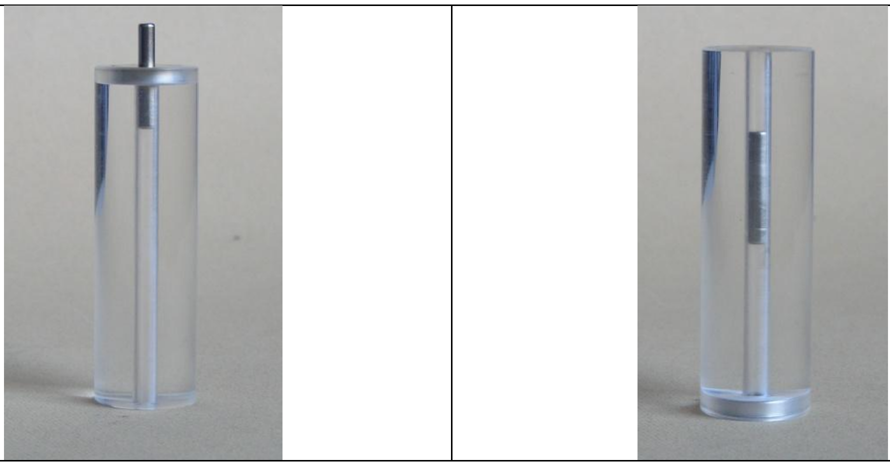

Figure 3. Photo of two magnets and a transparent hollow cylinder; the rod-magnet moves through the cylinder's hole.

Experimental setup (2nd problem)

The following items (to be used for the 2nd problem) are on your desk:

1. Press (together with a stone block); see separate instructions if needed
2. Scale (measures mass up to 5000 g, it has TARA function, see separate instructions if needed)
3. A transparent hollow cylinder with a ring-magnet embedded in its side.
4. One rod-magnet.
5. One narrow wooden stick (can be used to push the rod magnet out of the cylinder).

The setup is to be used as in Fig. 4 to measure the forces between the magnets. The upper plate of the press needs to be turned up-side-down in comparison to the first experimental problem. The narrow Aluminum rod is used to press the rod-magnet through the hollow cylinder. The scale measures the force (mass). The upper plate of the press can be moved downwards and upwards by using a wing nut. **Important: The wing nut moves 2mm when rotated 360 degrees.**

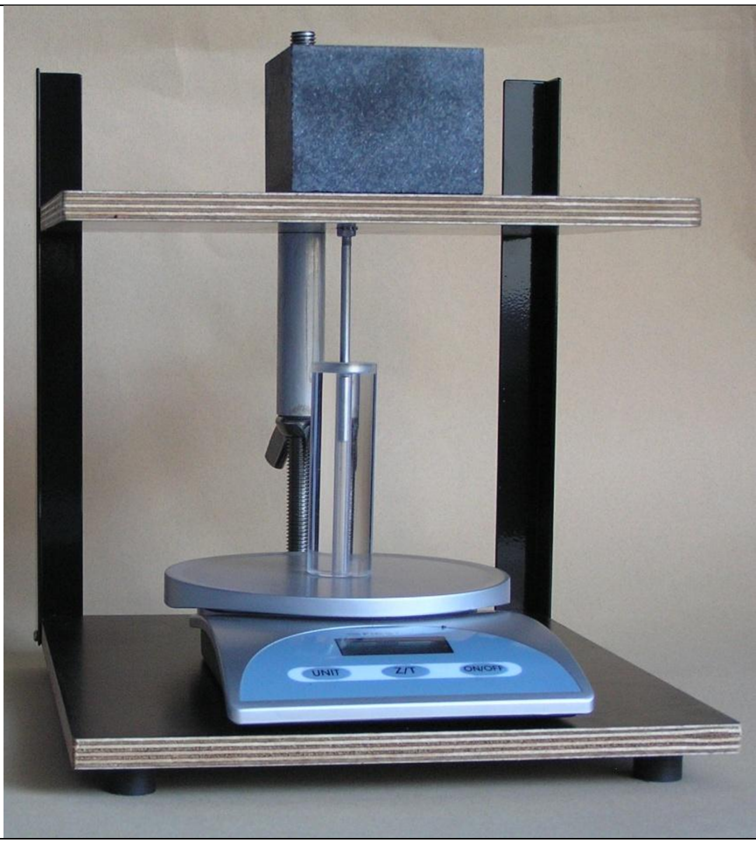

Figure 4. Photograph of the setup, and the way it should be used for measuring the force between the magnets.

Tasks

1. Determine qualitatively all equilibrium positions between the two magnets, assuming that the z - axis is positioned horizontally as in Fig. 2 , and draw them in the answer sheet. Label the equilibrium positions as stable (S)/unstable (U), and denote the like poles by shading, as indicated for one stable position in the answer sheet. You can do this Task by using your hands and a wooden stick. *(2.5 points)*
2. By using the experimental setup measure the force between the two magnets as a function of the z - coordinate. Let the positive direction of the z - axis point into the transparent cylinder (the force is positive if it points in the positive direction). For the configuration when the magnetic moments are parallel, denote the magnetic force by $F_{\uparrow\uparrow}(z)$, and when they are anti-parallel, denote the magnetic force by $F_{\uparrow\downarrow}(z)$. **Important: Neglect the mass of the rod-magnet (i.e., neglect gravity), and utilize the symmetries of the forces between magnets to measure different parts of the curves.** If you find any symmetry in the forces, write them in the answer sheets. Write the measurements on the answer sheets; beside every table schematically draw the configuration of magnets corresponding to each table (an example is given). *(3.0 points)*
3. By using the measurements from Task 2, use the millimeter paper to plot in detail the functional dependence $F_{\uparrow\uparrow}(z)$ for $z > 0$. Plot schematically the shapes of the curves $F_{\uparrow\uparrow}(z)$ and $F_{\uparrow\downarrow}(z)$ (along the positive and the negative z - axis). On each schematic graph

denote the positions of the stable equilibrium points, and sketch the corresponding configuration of magnets (as in Task 1). *(4.0 points)*

4. If we do not neglect the mass of the rod magnet, are there any qualitatively new stable equilibrium positions created when the z - axis is positioned vertically? If so, plot them on the answer sheet as in Task 1. *(0.5 points)*