Speed of light (solution)

In this document decimal comma is used instead of decimal point in graphs and tables

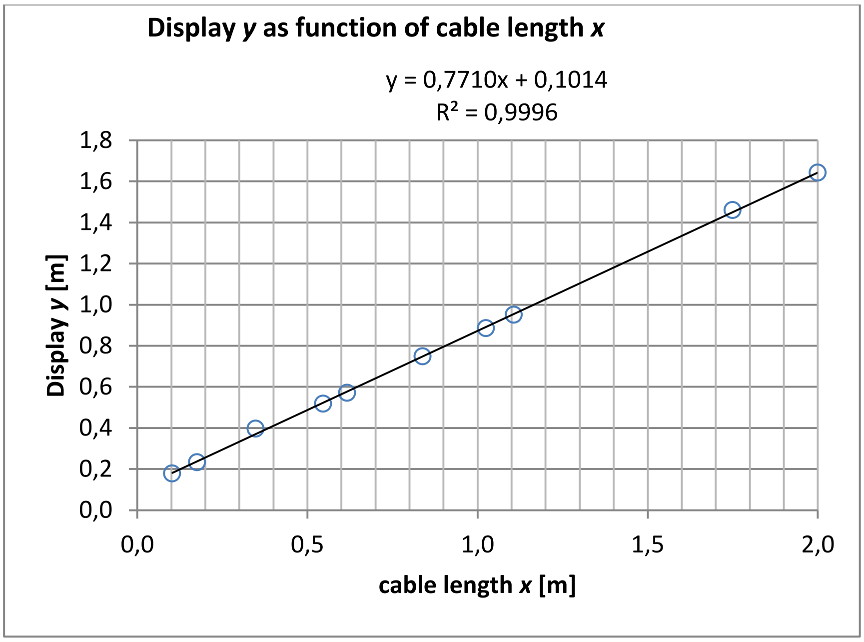
1.1	Use the LDM to measure the distance H from the top of the table to the floor. Write down the uncertainty ΔH. Show with a sketch how you perform this measurement.	0.4

$H = 907\text{ mm } \pm 2\text{ mm}$. See the sketch in the figure corresponding to 1.3b. It must appear how the height is measured with the LDM in the rear mode.

1.2a	Measure corresponding values of x and y. Set up a table with your measurements. Draw a graph showing y as a function of x.	1.8

Here, a 2 m cable is used, but 1 m is sufficient. There should be about 8 lengths evenly distributed in the interval from 0 m to 1 m.

x	y
m	m
0,103	0,177
0,176	0,232
0,348	0,396
0,546	0,517
0,617	0,570
0,839	0,748
1,025	0,885
1,107	0,950
1,750	1,459
2,000	1,642



1.2b	Use the graph to find the refractive index n_{co} for the material from which the core of the fiber optic cable is made. Calculate the speed of light v_{co} in the core of the fiber optic cable.	1.2

The refractive index is twice the gradient of the linear graph, $n_{co} = 2 \cdot 0.7710 \approx 1.54$.

The reason for that is that the travel time for a light pulse

$$t = \frac{x}{v_{co}} = \frac{x n_{co}}{c}$$

The display will therefore show $y = \frac{1}{2}ct + k \Leftrightarrow y = \frac{1}{2}n_{co}x + k$.

The speed of light in the core of the cable is $v_{co} = \frac{c}{n_{co}} = \frac{2{,}998\cdot 10^8 \frac{\text{m}}{\text{s}}}{2\cdot 0.7710} \approx 1.95 \cdot 10^8 \frac{\text{m}}{\text{s}}$

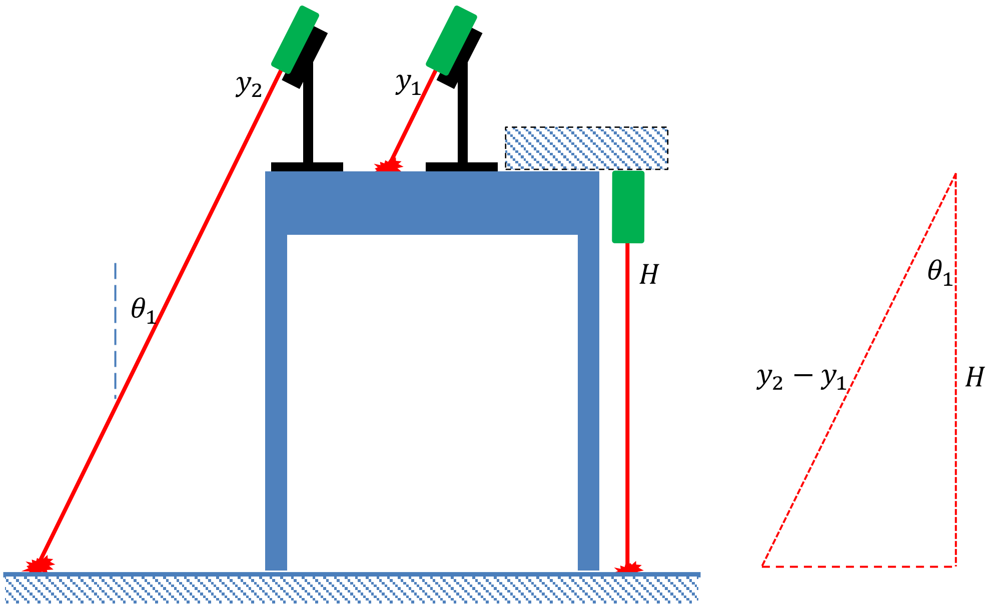
1.3a	Measure with the LDM the distance y_1 to the laser dot where the laser beam hits the table top. Then move the box with the LDM horizontally until the laser beam hits the floor. Measure the distance y_2 to the laser dot where the laser beam hits the floor. State the uncertainties.	0.2

$y_1 = 312\ \text{mm} \pm 2\ \text{mm}$, $y_2 = 1273\ \text{mm} \pm 2\ \text{mm}$

1.3b	Calculate the angle θ_1 using only these measurements y_1, y_2 and H (from problem 1.1). Determine the uncertainty $\Delta\theta_1$.	0.4

$$\theta_1 = \cos^{-1}\left(\frac{H}{y_2 - y_1}\right) = \cos^{-1}\left(\frac{907\ \text{mm}}{961\ \text{mm}}\right) = 19.30°$$

(see the figure)



Measuring the horizontal part of some triangle is very inaccurate because of the size of the laser dot. No marks will be awarded for that. Using $\delta = 2$ mm as the uncertainty of y_1, y_2 and H, the uncertainty of θ_1 can be calculated as follows:

$$\Delta \cos\theta_1 = \Delta\left(\frac{H}{y_2 - y_1}\right)$$

Using simple derivatives yields

$$\tan\theta_1 \cdot \Delta\theta_1 = \frac{\delta}{H} + \frac{2\delta}{y_2 - y_1}$$

$$\Delta\theta_1 = \frac{\left(\frac{\delta}{H} + \frac{2\delta}{y_2 - y_1}\right)}{\tan\theta_1} \cdot \frac{180°}{\pi} = \frac{\left(\frac{2}{907} + \frac{4}{961}\right)}{\tan 19{,}30°} \cdot \frac{180°}{\pi} \approx 1°$$

Otherwise, using min/max method

$$\Delta\theta_1 = \theta_{1\max} - \theta_1 = \cos^{-1}\left(\frac{H_{\min}}{y_{2\max} - y_{1\min}}\right) = \cos^{-1}\left(\frac{905\text{ mm}}{965\text{ mm}}\right) - \cos^{-1}\left(\frac{907\text{ mm}}{961\text{ mm}}\right) = 1.0°$$

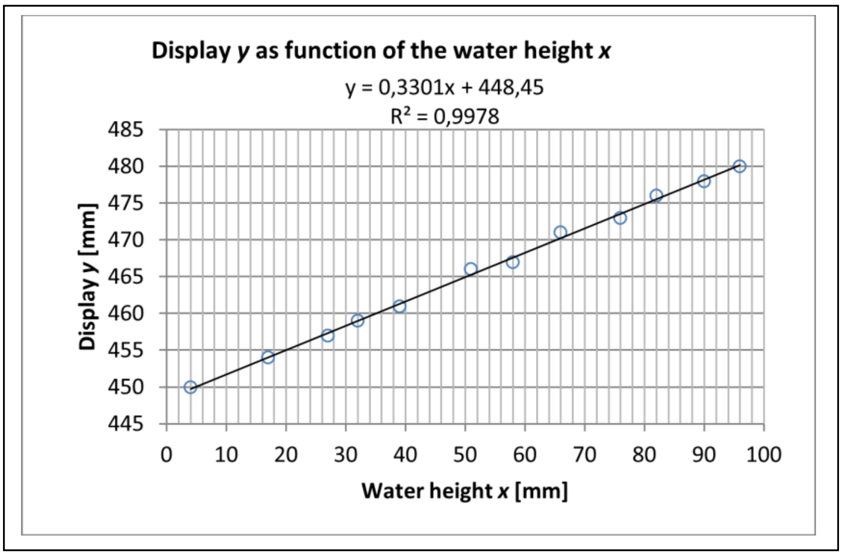
Alternatively, calculate $\Delta\theta_1$ using $\Delta(y_2 - y_1) = \sqrt{(\Delta y_1)^2 + (\Delta y_2)^2} = \sqrt{2}\delta$ and then

$$\tan\theta_1 \cdot \Delta\theta_1 = \sqrt{\left(\frac{\delta}{H}\right)^2 + \frac{2\delta^2}{(y_2 - y_1)^2}}$$

Also, accept $\delta = 1$ mm and $\Delta\theta_1 = 0.5°$.

1.4a	Measure corresponding values of x and y. Set up a table with your measurements. Draw a graph of y as a function of x.	1.6

x [mm]	y [mm]
4	450
17	454
27	457
32	459
39	461
51	466
58	467
66	471
76	473
82	476
90	478
96	480



1.4b	Use equations to explain theoretically what the graph is expected to look like.	1.2

The time it takes the light to reach the water surface is

$$t_1 = \frac{(h-x)/\cos\theta_1}{c}$$

From the water surface to the bottom the light uses the time

$$t_2 = \frac{x/\cos\theta_2}{v}$$

Total travel time forth and back

$$t = 2t_1 + 2t_2 = 2\frac{(h-x)/\cos\theta_1}{c} + 2\frac{x/\cos\theta_2}{v} = 2\frac{h-x}{c\cos\theta_1} + 2\frac{nx}{c\cos\theta_2}$$

Hence, the display will show (we simply write $n = n_{\mathrm{w}}$)

$$y = ½ct + k = \left(\frac{n}{\cos\theta_2} - \frac{1}{\cos\theta_1}\right)x + \frac{h}{\cos\theta_1} + k$$

which is a linear function of x. Then, using a trigonometric identity and Snell's law,

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\frac{\sin^2\theta_1}{n^2}}.$$

From this the gradient α is found to be

$$\alpha = \frac{n}{\sqrt{1-\frac{\sin^2\theta_1}{n^2}}} - \frac{1}{\cos\theta_1} = \frac{n^2}{\sqrt{n^2-\sin^2\theta_1}} - \frac{1}{\cos\theta_1}$$

1.4c	Use the graph to determine the refractive index n_{w} for water.	1.2

Knowing the gradient α from the graph, the index of refraction n is found by solving this equation. Introducing a practical parameter,

$$p = \alpha + \frac{1}{\cos\theta_1}$$

the above equation becomes

$$p = \frac{n_{\mathrm{w}}^2}{\sqrt{n_{\mathrm{w}}^2 - \sin^2\theta_1}} \Leftrightarrow n_{\mathrm{w}}^4 - p^2 n_{\mathrm{w}}^2 + p^2\sin^2\theta_1 = 0$$

with the solution

$$n_{\mathrm{w}} = \sqrt{\frac{p^2 \pm \sqrt{p^4 - 4p^2\sin^2\theta_1}}{2}} = \frac{\sqrt{2}}{2}p\sqrt{1 \pm \sqrt{1-\left(\frac{2\sin\theta_1}{p}\right)^2}}$$

From the graph is found $\alpha = 0.3301$, which leads to $p = 1.388356$ and hence

$$n_{\mathrm{w}} = 1.34676 \approx 1.347.$$

All solutions with $n_{\mathrm{w}} < 1$ are omitted.

Another and more elegant way of finding n_{w} is to use Snell's law in the equation

$$\alpha = \frac{n_{\mathrm{w}}}{\cos\theta_2} - \frac{1}{\cos\theta_1} = \frac{\sin\theta_1}{\sin\theta_2\cos\theta_2} - \frac{1}{\cos\theta_1} = \frac{2\sin\theta_1}{\sin 2\theta_2} - \frac{1}{\cos\theta_1}$$

This yields

$$\sin 2\theta_2 = \frac{2\sin\theta_1}{\alpha + \dfrac{1}{\cos\theta_1}}$$

From here θ_2 can be calculated leading to $n_{\mathrm{w}} = \frac{\sin\theta_1}{\sin\theta_2}$. This method also only uses the graph and the angle θ_1, and measurement of θ_2 is not involved).

The table value for pure water at normal conditions is $n_{\mathrm{w}} = 1.331$ at the wavelength $\lambda = 635$ nm.

The following approximations can be used: For small angles

$$n_{\mathrm{w}} \approx \frac{\sqrt{2}}{2} p \sqrt{1 + 1 - \frac{1}{2}\left(\frac{2\sin\theta_1}{p}\right)^2} \approx p\sqrt{1 - \left(\frac{\sin\theta_1}{p}\right)^2} \approx p\left(1 - \frac{1}{2}\left(\frac{\sin\theta_1}{p}\right)^2\right)$$

For very small angles, we get

$$n_{\mathrm{w}} \approx p \approx \alpha + 1$$

It is much simpler, but not recommendable, to do the experiment with very small $\theta_1 \approx 0$. Reflections in the water surface will ruin the signal from the bottom.

2.1 The dependence of the solar cell current on the distance to the light source

$$I(r) = \frac{I_a}{1 + \frac{r^2}{a^2}}$$

2.1a	Measure I as a function of r, and set up a table of your measurements.	1.0
2.1b	Determine the values of I_a and a by the use of a suitable graphical method.	1.0

slot #	r	I	$1/I$	r^2
	mm	mA	1/mA	mm^2
3	9.0	5.440	0.184	81
4	14.5	5.290	0.189	210
5	20.0	5.010	0.200	400
6	25.5	4.540	0.220	650
7	31.0	3.840	0.260	961
8	36.5	3.230	0.310	1332
9	42.0	2.730	0.366	1764
10	47.5	2.305	0.434	2256
11	53.0	1.985	0.504	2809
12	58.5	1.730	0.578	3422
13	64.0	1.485	0.673	4096
14	69.5	1.305	0.766	4830
15	75.0	1.140	0.877	5625
16	80.5	1.045	0.957	6480
17	86.0	0.930	1.075	7396
18	91.5	0.840	1.190	8372
19	97.0	0.755	1.325	9409
20	102.5	0.690	1.449	10506

$$I\left(1 + \frac{r^2}{a^2}\right) = I_a$$

$$r^2 = I_a a^2 \cdot \frac{1}{I} - a^2$$

$$a^2 = 12 \quad \mathrm{mm}^2 \pm 100\ \mathrm{mm}^2,$$

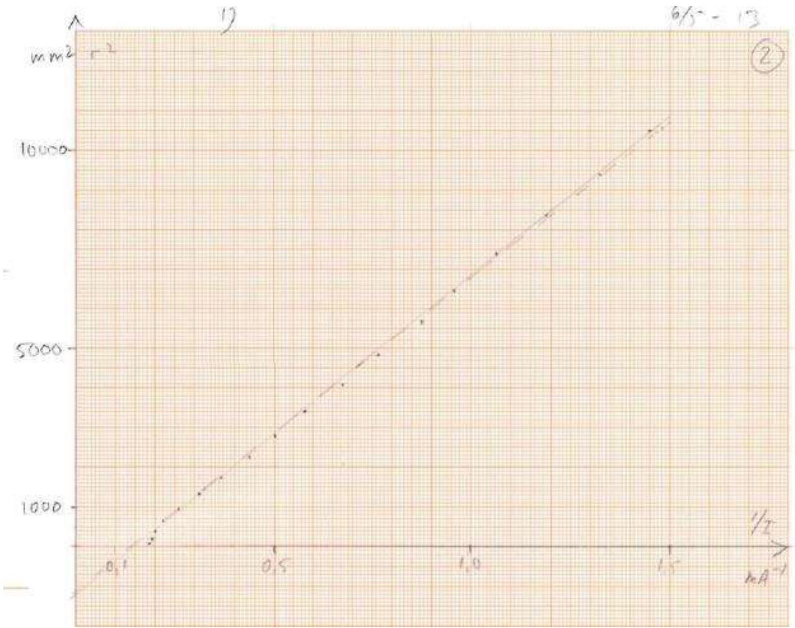
$$a = 35\ \mathrm{mm} \pm \pm 2\ \mathrm{mm}$$

$$I_a a^2 = \frac{10870-0}{1.50-0.15} \cdot \frac{\mathrm{mm}^2}{\mathrm{mA}^{-1}} = 8051.85 \ldots \mathrm{m} \quad {}^2\mathrm{mA}$$

$$I_a = \frac{8051.85 \frac{\mathrm{mm}^2}{\mathrm{mA}^{-1}}}{1200\ \mathrm{mm}^2} = 6.7\ \mathrm{mA} \pm 0.5\ \mathrm{mA}$$

$$(I_a a^2)_{\min} = \frac{10700-0}{1.50-0.14} \cdot \frac{\mathrm{mm}^2}{\mathrm{mA}^{-1}} = 7867.6 \ldots \mathrm{m} \quad {}^2\mathrm{mA}$$

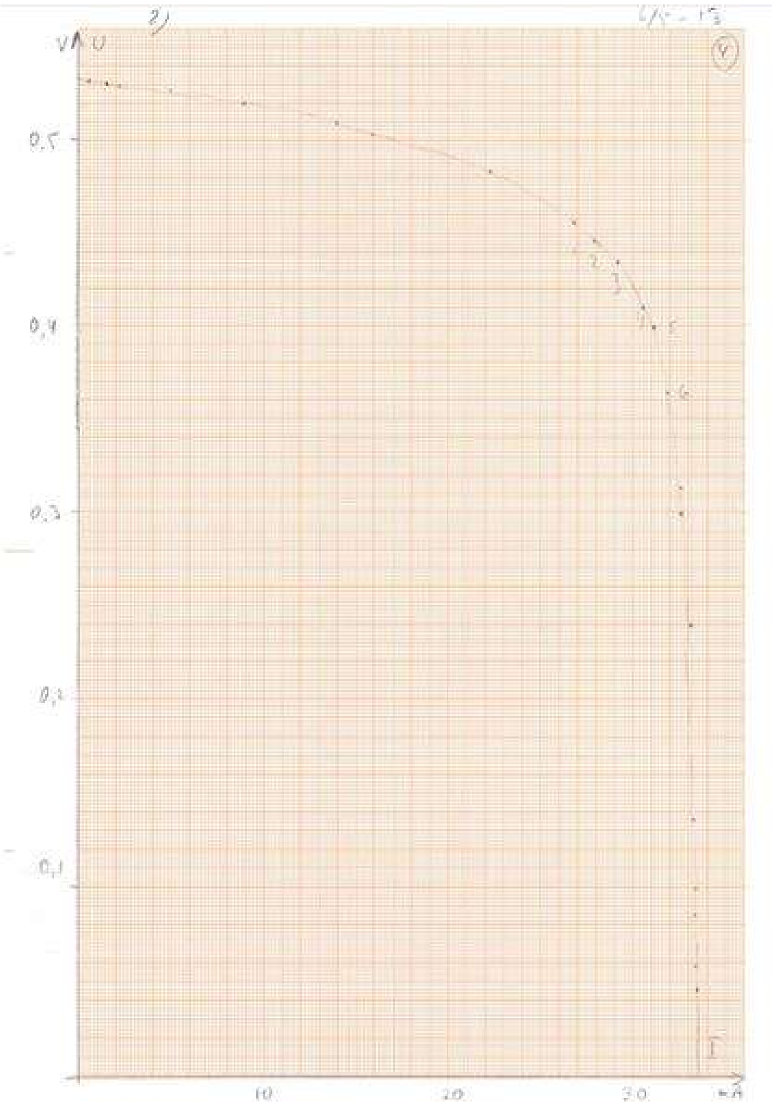
$$\rightarrow I_{a,\max} = \frac{(I_a a^2)_{\min}}{a^2{}_{\min}} = \frac{7867.6\ \mathrm{mm}^2\mathrm{mA}}{1100\ \mathrm{mm}^2} = 7.2\ \mathrm{mA}$$



2.2 Characteristic of the solar cell

2.2a	Make a table of corresponding measurements of U and I.	0.6
2.2b	Graph voltage as a function of current	0.8

I	U
mA	V
0.496	0.532
1.451	0.531
5.05	0.526
8.88	0.52
14.05	0.509
31.1	0.395
25.3	0.471
21.6	0.488
30.6	0.41
31.9	0.364
32.6	0.299
32.6	0.313
33.1	0.239
33.4	0.085
33.3	0.138
33.4	0.096
33.4	0.058
33.5	0.046
33.5	0.045
1.05	0.529
27.8	0.454
15.9	0.503
22.3	0.483
26.8	0.458
29.2	0.435



2.3 Theoretical characteristic for the solar cell

2.3a	Use the graph from question 2.2b to determine $I_{\max}$.	0.4
2.3b	Estimate the range of values of U for which the mentioned approximation is good. Determine graphically the values of I_0 and η for your solar cell.	1.2

$I = I_{\max}$ for $U = 0 \rightarrow I_{\max} = 33.5 \text{ mA}$

$\eta k_B T < 4 \cdot 1.381 \cdot 10^{-2} \text{ J/K} \cdot 300 \text{ K} = 0.103 \text{ eV}$

$$I = I_{\max} - I_0\left(\exp\left(\frac{eU}{\eta k_B T}\right) - 1\right) \approx I_{\max} - I_0 \exp\left(\frac{eU}{\eta k_B T}\right)$$

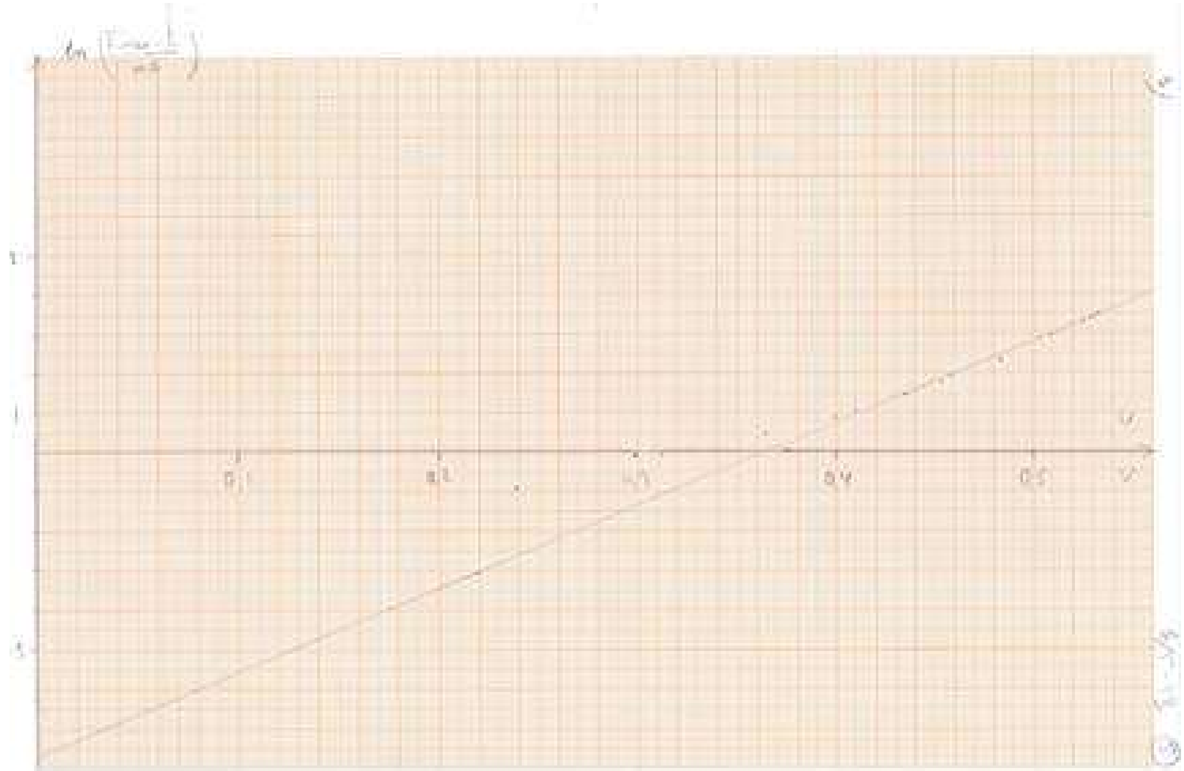
for $U > 0.4\ V$ where $\exp\left(\frac{eU}{\eta k_B T}\right) > \exp(4) \gg 1$

$$\ln\left(\frac{I_{\max} - I}{\text{mA}}\right) = \frac{e}{\eta k_B T} U + \ln\left(\frac{I_0}{\text{mA}}\right)$$

$$\frac{e}{\eta k_B T} = \frac{4.03 - (-7.7)}{0.56 \text{ V}} = 20.95 \text{ V}^{-1}$$

$$I_0 = e^{-7.7} \text{mA} = 0.45\ \mu A$$

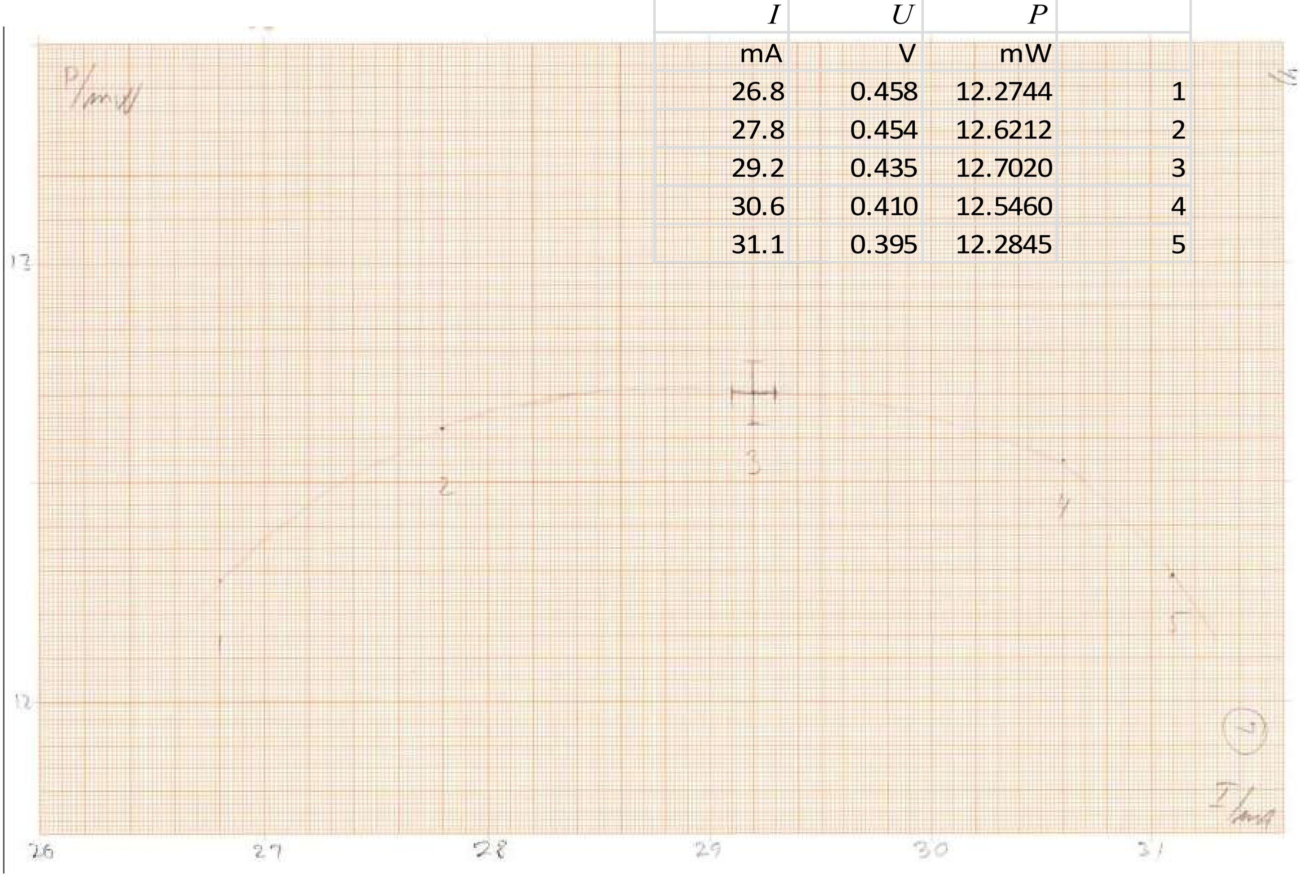
$$\rightarrow \boldsymbol{\eta} = \frac{e/(k_B T)}{20.95 \text{ V}^{-1}} = 1.85$$



2.4 Maximum power for a solar cell

2.4a	The maximum power that the solar cell can deliver to the external circuit is denoted $P_{\max}$. Determine $P_{\max}$ for your solar cell through a few, suitable measurements. (You may use some of your previous measurements from question 2.2)	0.5
2.4b	Estimate the optimal load resistance R_{opt}, i.e. the total external resistance when the solar cell delivers its maximum power to R_{opt}. State your result with uncertainty and illustrate your method with suitable calculations.	0.5

I	U	P	
mA	V	mW	
26.8	0.458	12.2744	1
27.8	0.454	12.6212	2
29.2	0.435	12.7020	3
30.6	0.410	12.5460	4
31.1	0.395	12.2845	5



$P_{\max} = (12.7 \pm 0.1)\text{mW at } I = (28.8 \pm 0.2)\text{mA}$

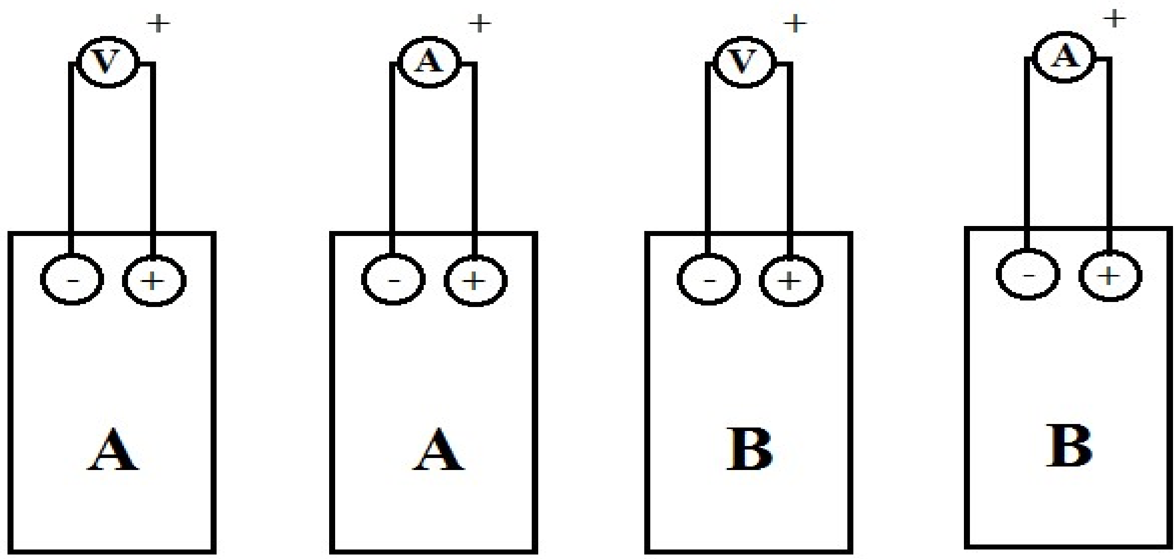
$$R_{\text{opt}} = \frac{P_{\max}}{I_{\text{opt}}^2} = \frac{12.71\,\text{mW}}{(28.8\,\text{mA})^2} = (15.3 \pm 0.3)\Omega$$

2.5 Comparing the solar cells

2.5a	Measure, for the given illumination: - The maximum potential difference U_A that can be measured over solar cell A. - The maximum current I_A that can be measured through solar cell A. Do the same for solar cell B.	0.5
2.5b	Draw electrical diagrams for your circuits showing the wiring of the solar cells and the meters.	0.3

2.5a. U_A=0.512 V I_A=16.465 mA U_B=0.480 V I_B = 16.325 mA

2.5b.



2.6 Couplings of the solar cells

2.6	Determine which of the four arrangements of the two solar cells yields the highest possible power in the external circuit when one of the solar cells is shielded with the shielding plate (J in Fig. 2.1). Draw the corresponding electrical diagram.	1.0

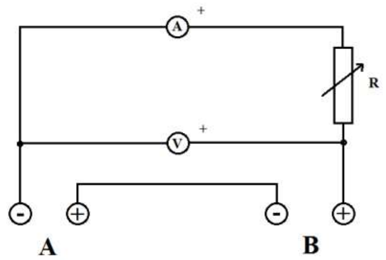
Two approaches:

Approach 1: use a constant setting of the variable resistor to simulate a constant external load.

Approach 2: use the hint given in the question and measure values of maximal U and maximal I independently (no variable resistor involved).

In the following only measurements for approach 1 are presented.

a.



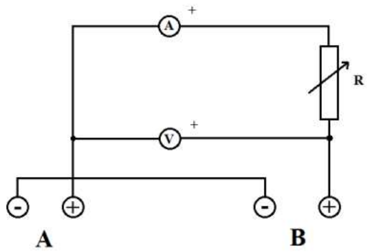
Unshielded (adjusting R for reasonable P)

13.10 mA; 0.794 V; 10.4 mW

A shielded: 0.37 mA; 0.022 V

B shielded: 0.83 mA; 0.049 V

b.

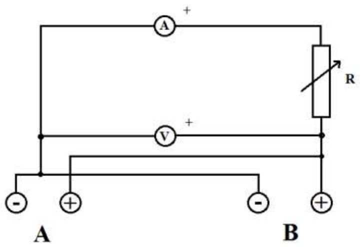


R like in a.

A shielded: 1.47 mA; 0.088 V

B shielded: -2.82 mA; -0.170 V

c.

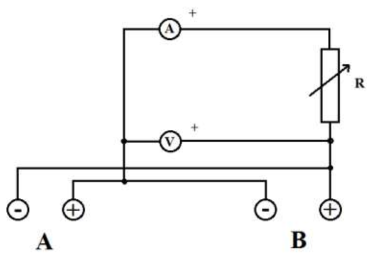


R like in a.

A shielded: 6.89 mA; 0.415 V

B shielded: 6.905 mA; 0.4165 V

d.



R like in a.

A shielded: 7.14 mA; 0.436 V

B shielded: -7.76 mA; -0.474 V

Conclusion: Best power: Set-up d with B shielded. (Solar cell A slightly better than B).

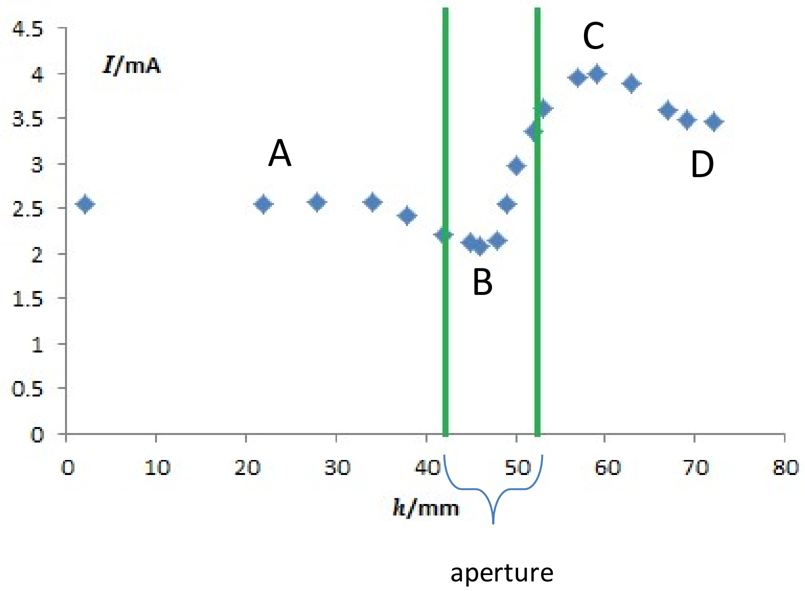
(2.7 on next page)

2.7 The effect of the optical vessel (large cuvette) on the solar cell current

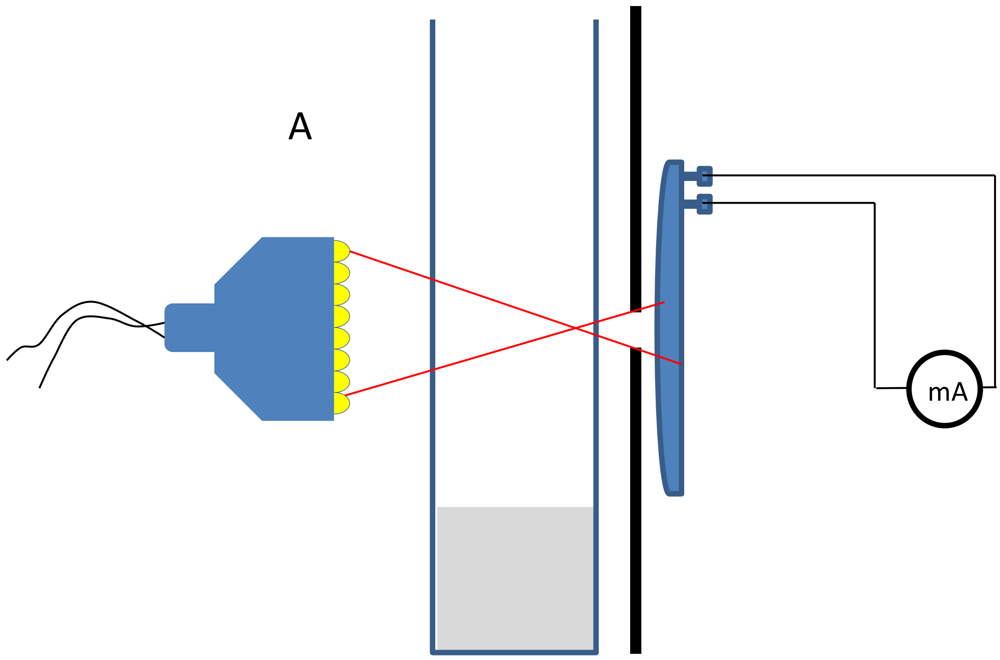
2.7a	Measure the current I, now as a function of the height, h, of water in the vessel, see Fig. 2.8. Make a table of the measurements and draw a graph.	1.0
2.7b	Explain with only sketches and symbols why the graph looks the way it does.	1.0
2.7c	For this set-up do the following: - Measure the distance r_1 between the light source and the solar cell, and the current I_1. - Place the empty vessel immediately in front of the circular aperture and measure the current I_2. - Fill up the vessel with water, almost to the top, and measure the current I_3.	0.6
2.7d	Use your measurements from 2.7c to find a value for the refractive index n_w for water. Illustrate your method with suitable sketches and equations. You may include additional measurements.	1.6

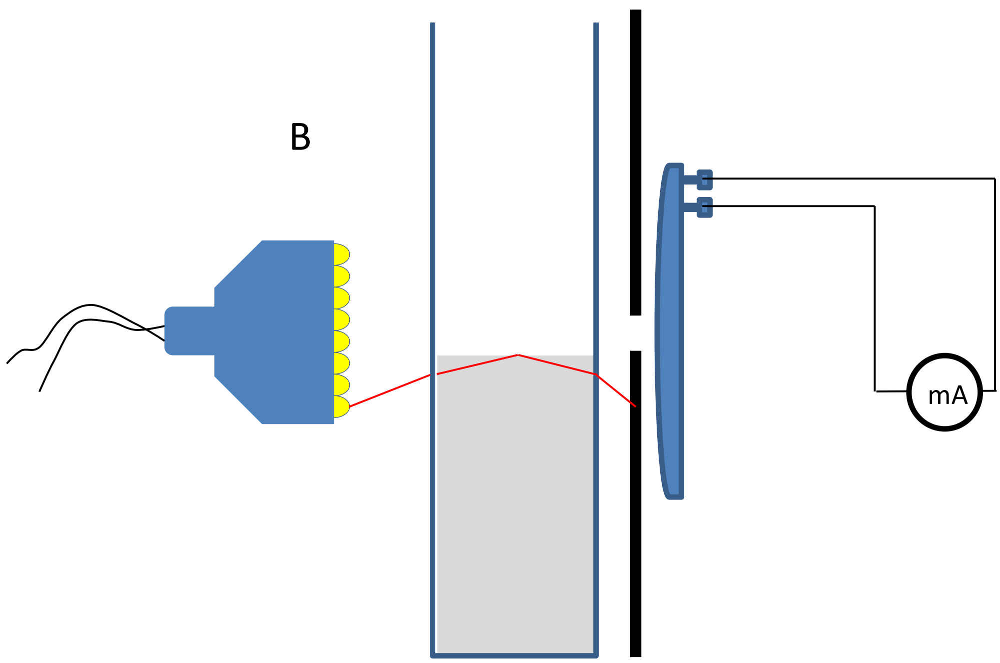
2.7a

h mm	I mA
2	2.54
22	2.55
28	2.56
34	2.57
38	2.42
42	2.21
45	2.13
46	2.08
48	2.15
49	2.54
50	2.97
52	3.36
53	3.61
57	3.96
59	3.99
63	3.89
67	3.6
69	3.49
72	3.47

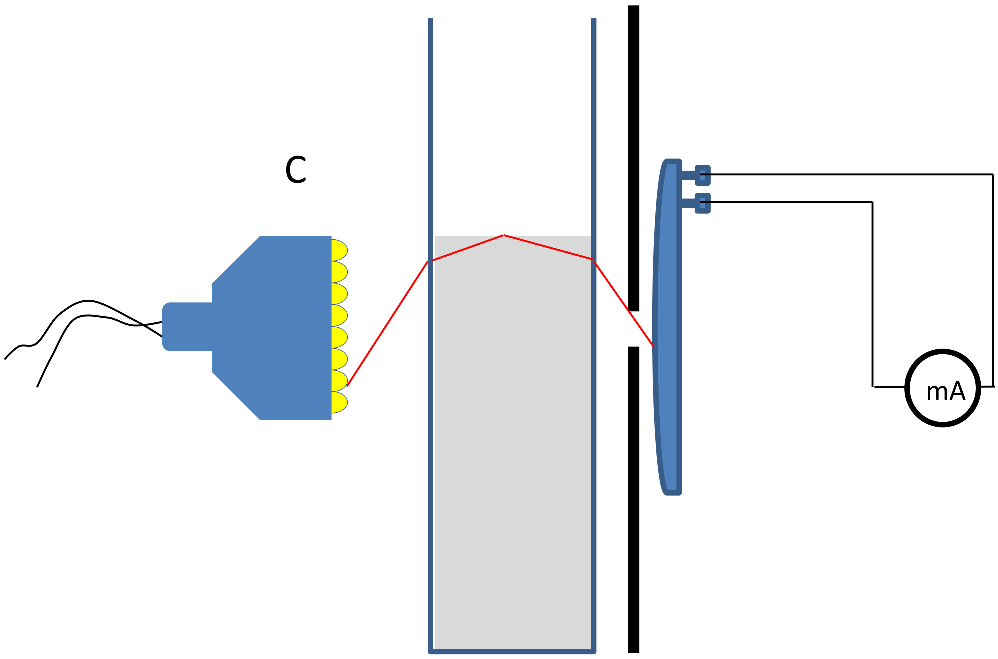


2.7b ***Exemple*** drawings for position A, B, C and D on previous graph:

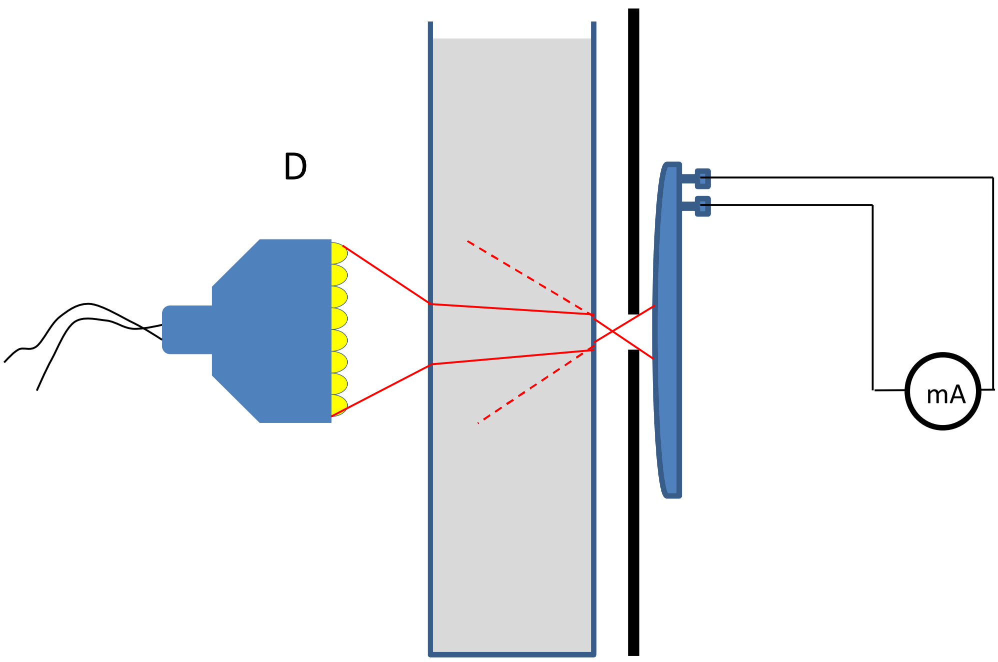




C

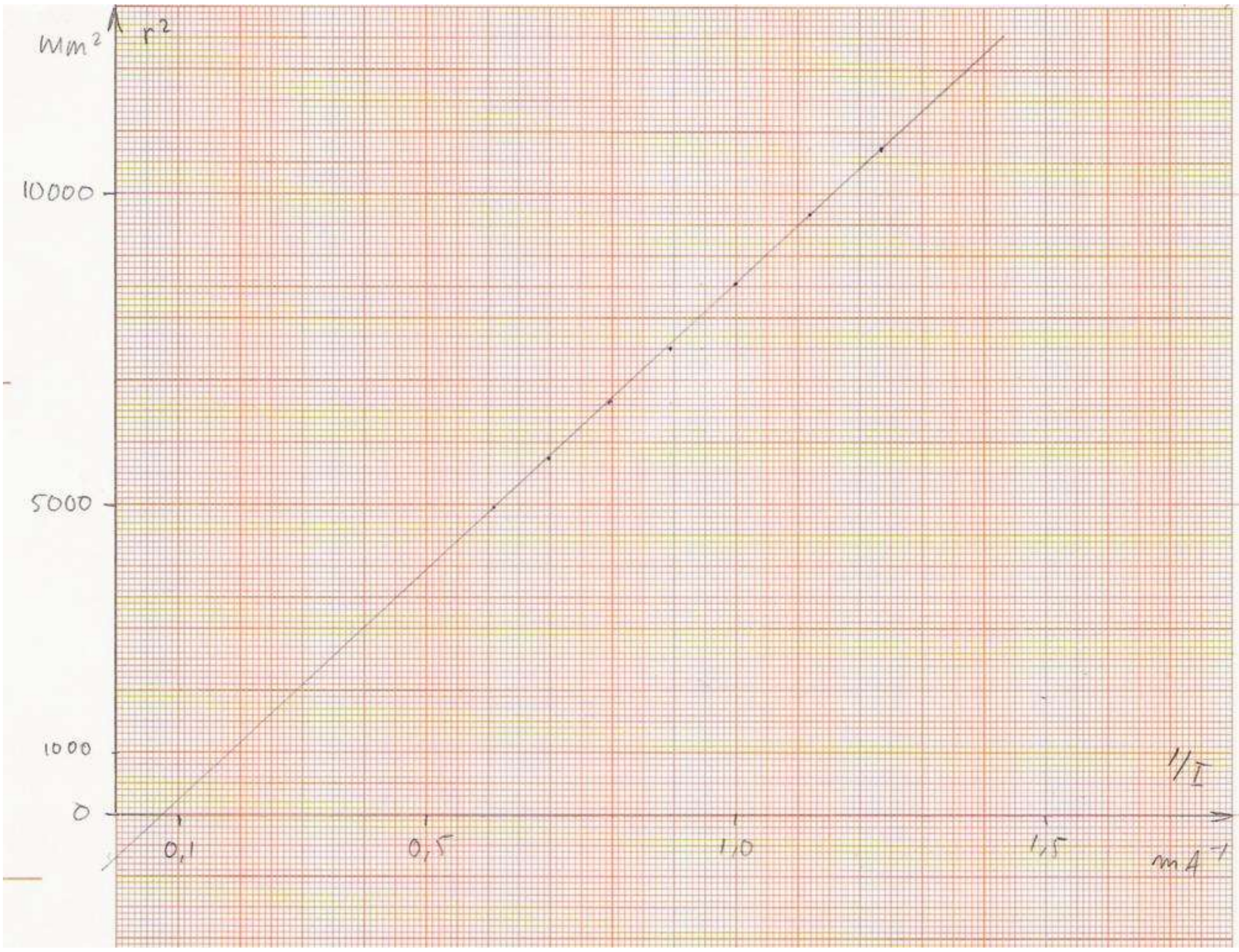


D

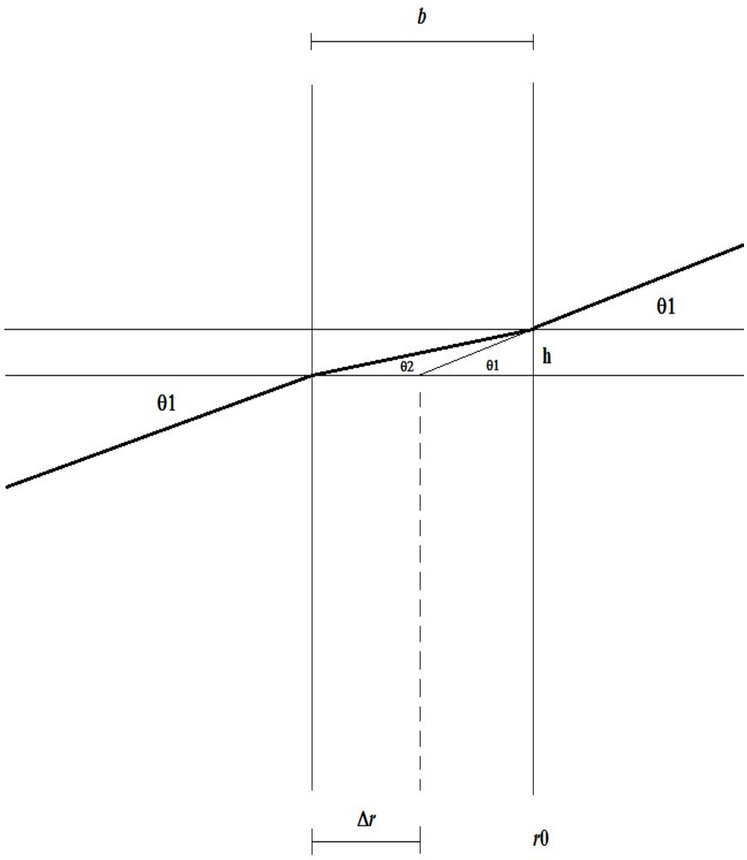


2.7c NOTE: The exemplar measurements are from a different lamp than in 2.1. For a solution to 2.7d using the distance graph it is necessary to refer to the graph below.
$r_1 = 103.5$ mm; $I_1 = 0.8$ mA; $I_2 = 0.70$ mA; $I_3 = 0.85$ mA

$$\frac{1}{I_3} \cdot \frac{I_2}{I_1} = 1.02 \quad \text{mA}^{-1} \sim r_c^2 = 880 \quad \text{mm}^2 \sim r_c = 93.8 \text{ mm}$$



2.7d



$$h = (b - \Delta r)\tan\theta_1 = b\tan\theta_2 \Rightarrow \frac{b}{b-\Delta r} = \frac{\tan\theta_1}{\tan\theta_2} \approx \frac{\sin\theta_1}{\sin\theta_2} = n \text{, da } \theta_2 < \theta_1 << 1.$$

$$n_w \approx \frac{b}{b - \Delta r} = \frac{b}{b - (r_1 - r_c)} = \frac{26.0\text{ mm}}{26.0\text{ mm} - (103.5 - 93.8)\text{mm}} = 1.6$$

NOTE: Better results may be obtained. The uncertainty is rather large in this method because of the subtraction of two large numbers for Δr

A different method is to determine the shift by actually moving the set-up and perhaps making an interpolation in directly measured data.