Diffraction due to Helical Structure[1]

(Total marks: 10)

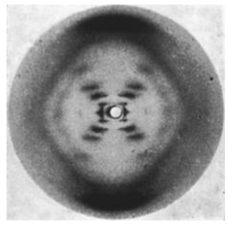


Figure 1: Photo 51

Introduction

The X-ray diffraction image of DNA (Fig. 1) taken in Rosalind Franklin's laboratory, famously known as "Photo 51", became the basis of the discovery of the double helical structure of DNA by Watson and Crick in 1952. This experiment will help you understand diffraction patterns due to helical structures using visible light.

Objective

To determine geometrical parameters of helical structures using diffraction.

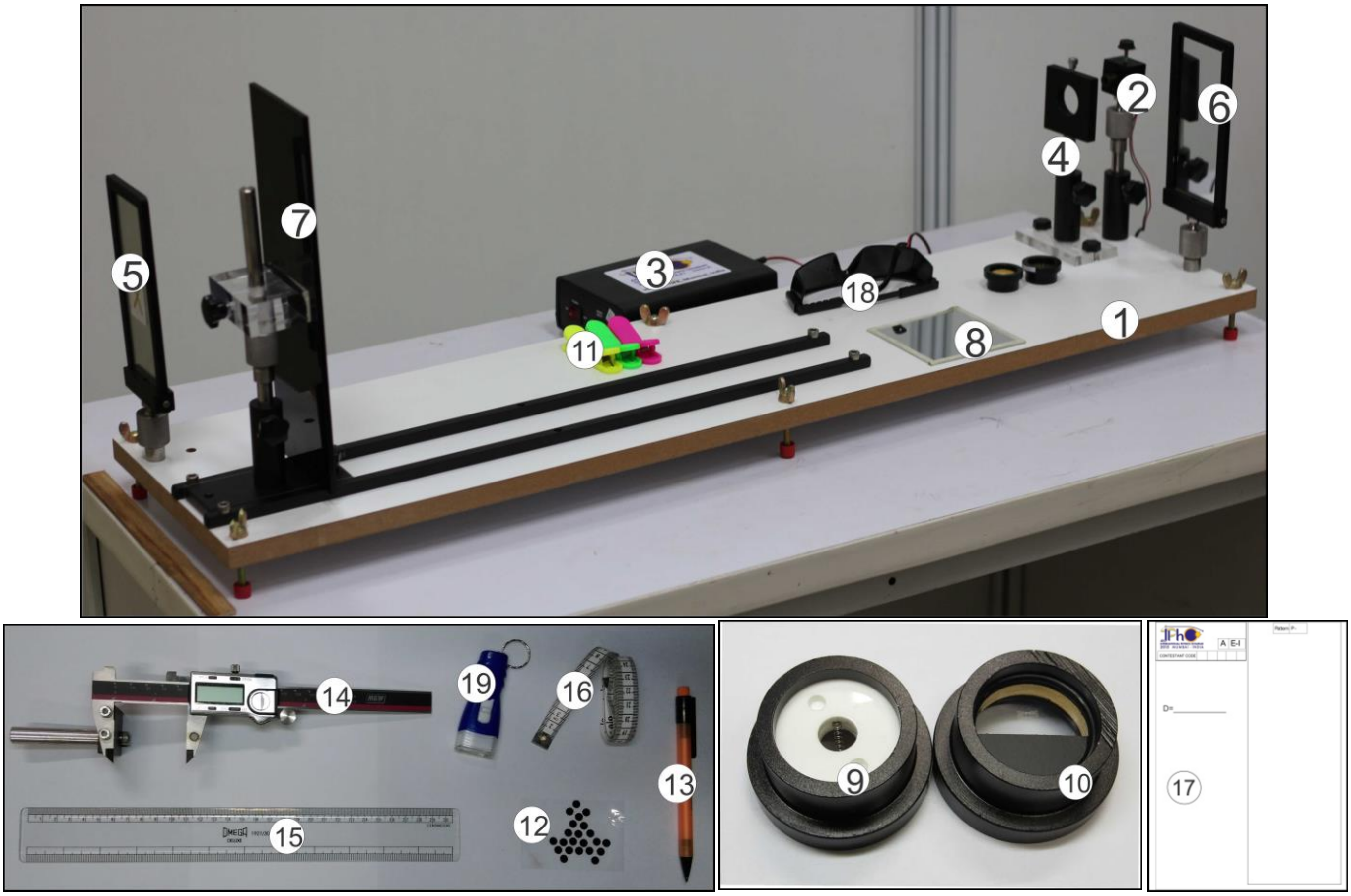


Figure 2: Apparatus for E-I

List of apparatus

[1]	Wooden platform	[11]	Plastic clips
[2]	Laser source with its mount and base	[12]	Circular black stickers
[3]	DC regulated power supply for the Laser source	[13]	Mechanical pencil
[4]	Sample holder with its base	[14]	Digital caliper with a mount
[5]	Left reflector (front coated mirror)	[15]	Plastic scale (30 cm)
[6]	Right reflector (front coated mirror)	[16]	Measuring tape (1.5 m)
[7]	Screen (10 cm x 30 cm) with its mount and base	[17]	Pattern marking sheets
[8]	Plane mirror (10 cm x 10 cm)	[18]	Laser safety goggles
[9]	Sample I (helical spring)	[19]	Battery operated flashlight
[10]	Sample II (double-helix-like pattern printed on glass plate)		

Note: Items [1], [3], [14], [15], [16] and [18] are also used in experiment E-II.

[1] *Praveen Pathak (HBCSE-TIFR, Mumbai), Charudatt Kadolkar (IIT, Guwahati), and Manish Kapoor (Christ Church College, Kanpur) were the principal authors of this problem. The contributions of the Academic Committee, Academic Development Group and the International Board are gratefully acknowledged.*

Description of apparatus

Wooden platform [1]: A pair of guiding rails, laser, reflectors, screen and sample mounts are rigidly fixed on it.
Laser source with its mount and base [2]: Laser source of wavelength $\lambda = 635$ nm (1 nm $= 10^{-9}$ m) is fixed in a metallic mount clamped to the base using a ball joint ([20] in Fig. 3) allowing the adjustment in X-Y-Z directions. The laser body can be rotated and clamped using the top lock-in screw. The beam focus can be adjusted by rotating the front lens cap (red arrow in Fig. 3) to obtain a clear and sharp diffraction pattern.
DC regulated power supply [3]: The front panel has an intensity switch (high/low), socket for laser source connector and three USB sockets. The back panel has power switch and mains power socket (inset of Fig. 4).

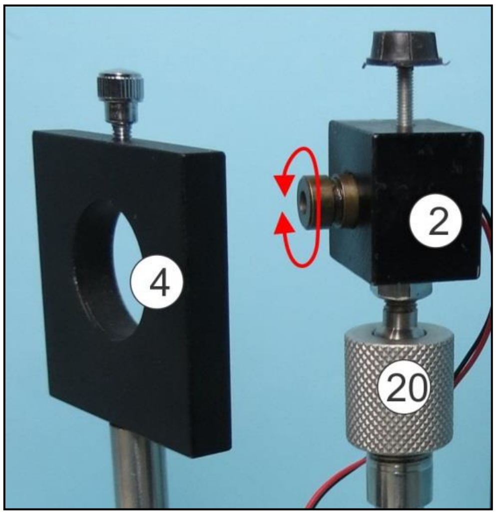


Figure 3: Laser source and sample holder. [20] Ball joint.

Figure 4: DC regulated power supply

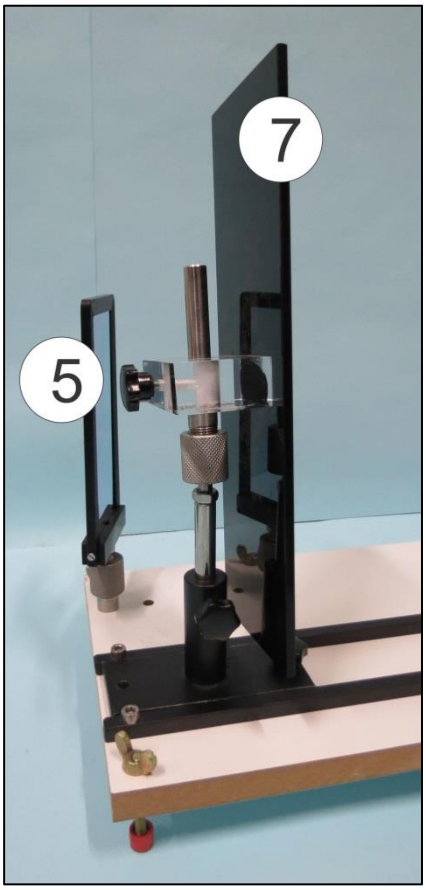


Figure 5: Left reflector and screen

Sample holder with its base [4]: Use the top locking screw to affix the samples in it (Fig. 3). The sample holder can be adjusted horizontally, vertically and rotated.
Left reflector [5]: This reflector is fixed to the platform (Fig. 5). Do not use the side marked X.
Right reflector [6]: This reflector is fixed to the platform and is removable (It will be removed in experiment E-II). Do not use the side marked X.
Screen with its mount [7]: The screen is mounted on ball joint and a base allowing rotational adjustments in all directions (Fig 5). The screen can be fixed as shown in Fig. 2 or Fig. 6 as required.

Sample I [9]: A helical spring fixed on a circular mount using white acrylic plates.

Sample II [10]: A double-helix-like pattern printed on a glass plate which is fixed on a circular mount.

Digital caliper with a mount [14]: Digital caliper is fixed to a mount (the mount is required in E-II). It has an On/Off switch, a switch to reset the reading to zero, a mm/inch selector (keep on mm), a locking screw and a knob for moving the right jaw. The digital caliper can be used to make measurements on pattern marking sheets.

Pattern marking sheets [17]: The given pattern marking sheets can be folded in half and clipped onto the screen using the plastic clips. Ensure that you mark the diffraction pattern within the rectangular box.

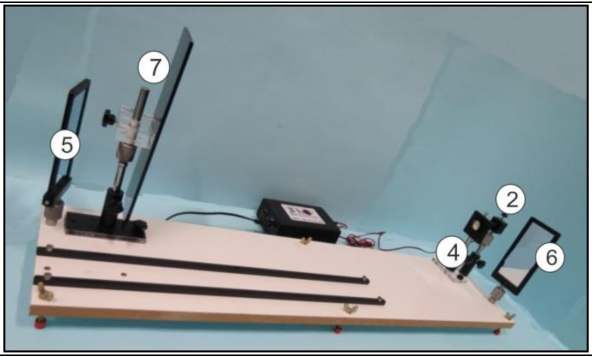


Figure 6: Alternate position of screen compared to that shown in Fig. 2

Theory

A laser beam of wavelength λ, falling normally on a cylindrical wire of diameter a, is diffracted in the direction perpendicular to the wire. The resulting intensity pattern as observed on a screen is shown in Fig. 7.

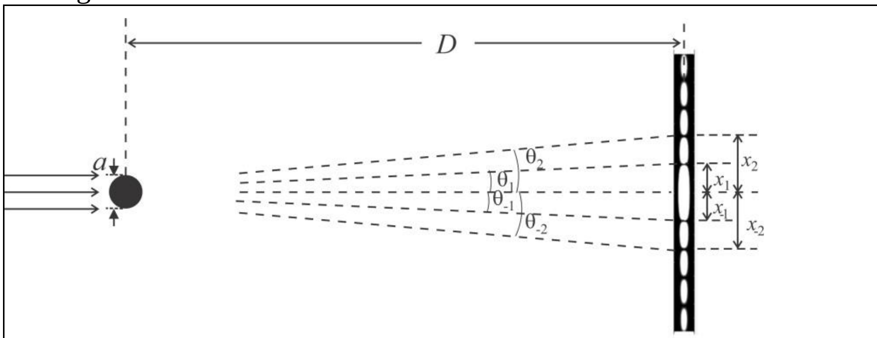


Figure 7: Schematic of the diffraction pattern due to single cylindrical wire of diameter a.

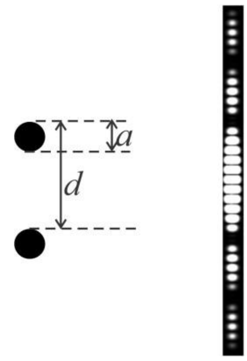


Figure 8: Schematic of diffraction pattern due to two cylindrical wires

The intensity distribution as a function of angle θ with the incident direction is given by

$$I(\theta) = I(0)\left[\frac{\sin\beta}{\beta}\right]^2 \qquad \text{where } \beta = \frac{\pi a \sin\theta}{\lambda}$$

The central spot is bright and for other angles, when $\sin\beta$ ($\beta \neq 0$) is zero, the intensity vanishes. Thus the intensity distribution has n^{th}minimum at the angle θ_n, given by

$$\sin\theta_n = \pm n\frac{\lambda}{a} \qquad n = 1, 2, 3, 4, 5 \ldots.$$

Here $\pm$ refers to both sides of the central spot ($\theta = 0$).

The diffraction pattern due to two parallel identical wires kept at a distance d from each other (Fig. 8) is a combination of two patterns (diffraction due to a single wire and interference due to two wires). The resultant intensity distribution is given by,

$$I(\theta) = I(0)\cos^2\delta\left[\frac{\sin\beta}{\beta}\right]^2$$

where $\delta = \frac{\pi d \sin\theta}{\lambda}$ and $\beta = \frac{\pi a \sin\theta}{\lambda}$

For a screen placed at a large distance D from the wire, the positions of the minima on the screen are observed at $x_{\pm n} = \pm n \frac{\lambda D}{a}$ due to diffraction and at $x_{\pm m} = \pm\left(m - \frac{1}{2}\right)\frac{\lambda D}{d}$ due to interference (where $m, n = 1, 2, 3, 4, 5 \ldots$). Similarly for a set of four identical wires (Fig. 9), the net intensity distribution is a combination of diffraction from each wire and interference due to pairs of wires and hence depends on a, d and s. In other words, the combination of three different intensity patterns is observed.

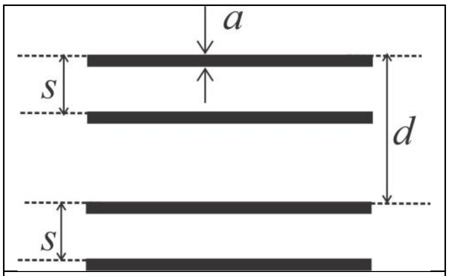


Figure 9: Set of four wires

Initial adjustments

1. Switch on the laser source and adjust both reflectors so that the laser spot falls on the screen.
2. Use the plastic scale and adjust the laser mount and reflectors such that the laser beam is parallel to the wooden platform.
3. Make sure that the laser spot falls near the centre of the screen.
4. Switch off the laser source. Clamp the pattern marking sheet on the screen.
5. Clamp the given plane mirror on the screen using plastic clips and switch on the laser again.
6. Adjust the screen so that the laser beam retraces its path back to the laser source. Remove the mirror once your alignment is completed.
7. Lights in the cubicle may be switched on/off as required.

Experiment

Part A: Determination of geometrical parameters of a helical spring[2]

Sample I is a helical spring of radius R and pitch P made of a wire of uniform thickness a_1 as shown in Fig. 10(a). When viewed at normal incidence its projection is equivalent to two sets of parallel wires of the same thickness separated by distance d_1 and angle $2\alpha_1$ between them (Fig. 10(b)).

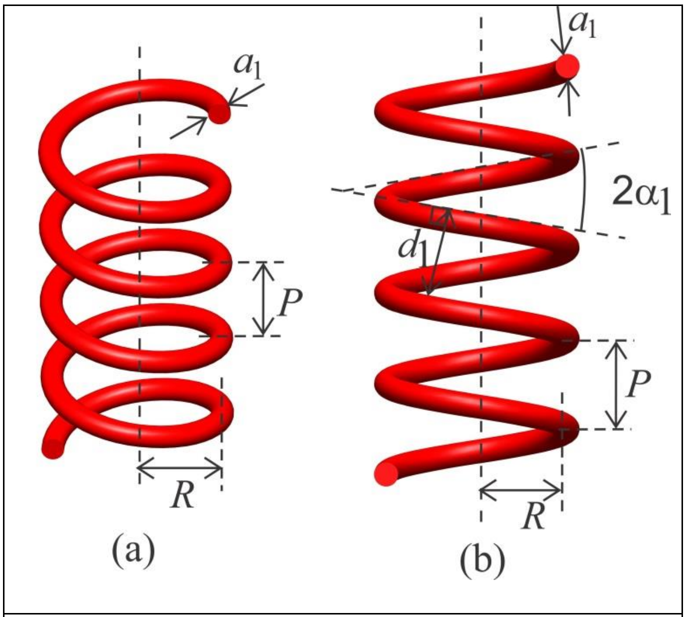


Figure 10: (a) Typical view of helical spring (b) Schematic diagram when viewed at normal incidence

[2] Reference: G. Braun, D. Tierney and H. Schmitzer, Phys. Teach. **49**, 140 (2011).

- Mount sample I in the sample holder ensuring that the spring is vertical.
- Obtain a clear and sharp X-shaped diffraction pattern on the pattern marking sheet.
- For this you may adjust
  - laser beam focus (rotate lens cap)
  - beam orientation (rotate the laser body so that only two turns of the spring are illuminated)
  - laser intensity (high/low switch on power supply)
  - ambient light (by switching on or off cubicle light)

If the central maximum is very bright, you may stick circular black stickers on the pattern marking sheet to reduce scattering.

Tasks	Description	Marks
A1	Mark the appropriate positions (using given pencil [13]) of the intensity minima to determine a_1 and d_1 on the both sides of the central spot on the pattern-marking sheet. Please label your pattern-marking sheets as P-1, P-2 etc.	**0.7**
A2	Measure the appropriate distances using digital calipers and record them in Table A1 for determining a_1.	**0.5**
A3	Plot a suitable graph, label it Graph A1 and from the slope, determine a_1.	**0.7**
A4	Measure the appropriate distances and record them in Table A2 for determining d_1.	**0.8**
A5	Plot a suitable graph, label it Graph A2 and from the slope, determine d_1.	**0.6**
A6	From the X-shaped pattern, determine the angle α_1.	**0.2**
A7	Express P in terms of d_1 and α_1 and calculate P.	**0.2**
A8	Express R in terms of P and α_1 and calculate R (neglect a_1).	**0.2**

Part B: Determination of geometrical parameters of double-helix-like pattern

Figure 11(a) shows two turns of a double helix. Fig. 11(b) is a two-dimensional projection of this double helix when viewed at normal incidence. Each helix of thickness a_2 has an angle $2\alpha_2$ and perpendicular distance d_2 between turns. The separation between two helices is s. Sample II is a double-helix-like pattern printed on glass plate (Fig. 12), whose diffraction pattern is similar to that of a double helix. In this part, you will determine the geometrical parameters of sample II.

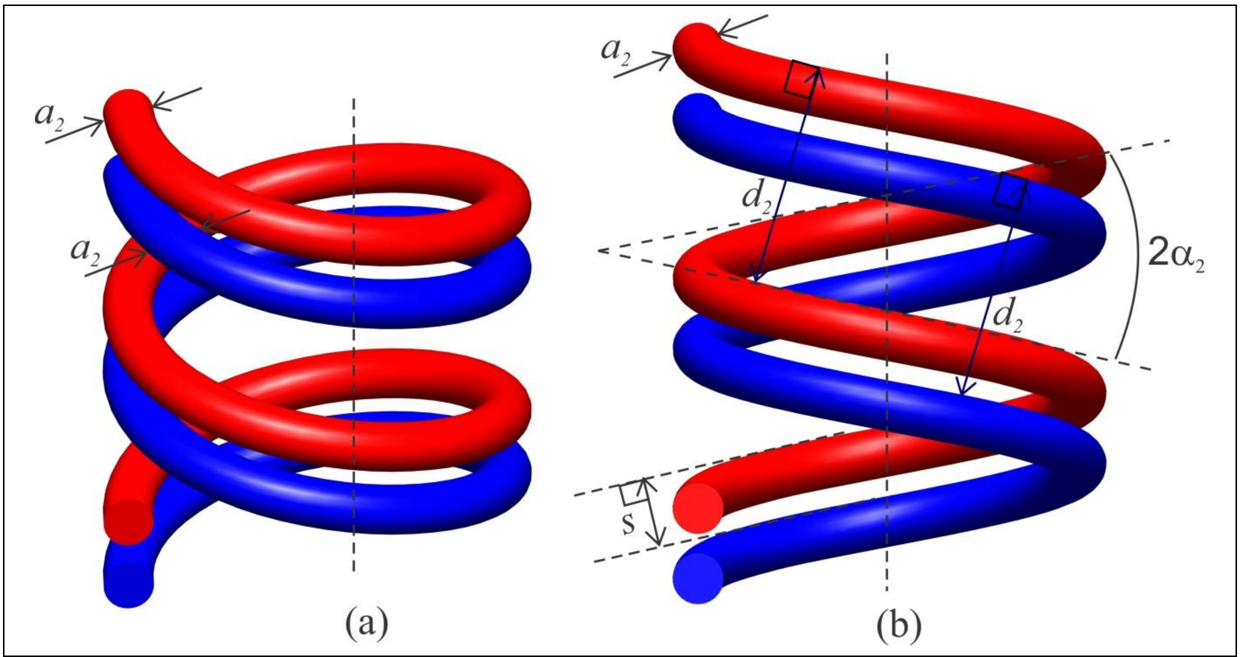


Figure 11: (a) Typical view of double-helical spring (b) Its schematic diagram when viewed at normal incidence.

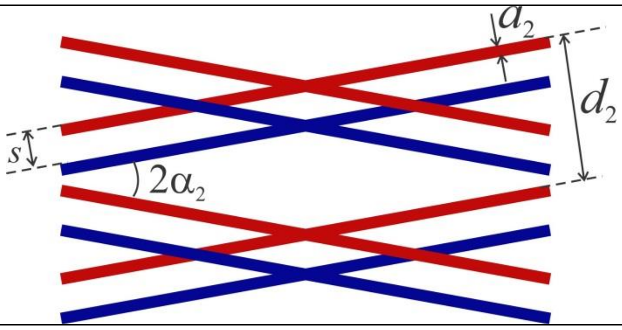


Figure 12: Double-helix-like pattern of sample II

- Mount the sample II in sample holder.
- Attach a new pattern-marking sheet on the screen.
- Obtain clear and sharp X-shaped diffraction pattern on the screen.

Tasks	**Description**	**Marks**
B1	Mark the appropriate positions of the minima on either side of the central spot to determine a_2, s and d_2. You can use more than one pattern marking sheets.	**1.1**
B2	Measure the appropriate distances and record them in Table B1 for determining a_2	**0.5**
B3	Plot suitable graph, label it Graph B1 and from the slope, determine a_2.	**0.5**
B4	Measure the appropriate distances and record them in Table B2 for determining s.	**1.2**
B5	Plot suitable graph, label it Graph B2 and from the slope determine s.	**0.5**
B6	Measure the appropriate distances and record them in Table B3 for determining d_2	**1.6**
B7	Plot suitable graph, label it Graph B3 and from the slope, determine d_2.	**0.5**
B8	From the X-shaped pattern, determine the angle α_2.	**0.2**

Diffraction due to surface tension waves on water[1]

Introduction

Formation and propagation of waves on a liquid surface are important and well-studied phenomena. For such waves, the restoring force on the oscillating liquid is partly due to gravity and partly due to surface tension. For wavelengths much smaller than a critical wavelength, λ_c, the effect of gravity is negligible and only surface tension effects need be considered ($\lambda_c = 2\pi\sqrt{\frac{\sigma}{\rho g}}$, where σ is the surface tension, ρ is the density of the liquid and g is the acceleration due to gravity).

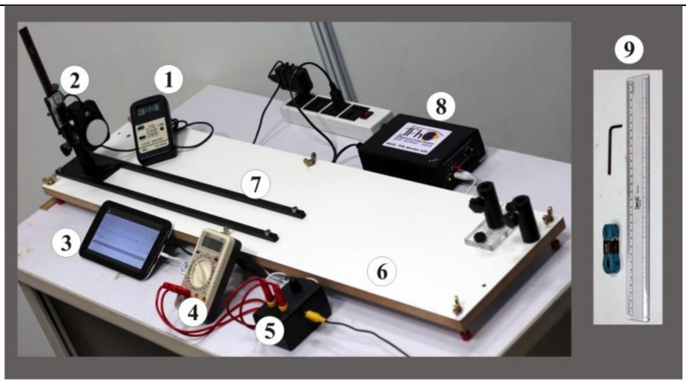
In this part, you will study surface tension waves on the surface of a liquid, which have wavelengths smaller than λ_c. Surface tension is a property of liquids due to which the liquid surface behaves like a stretched membrane. When the liquid surface is disturbed, the disturbance propagates as a wave just as on a membrane. An electrically-driven vibrator is used to produce waves on the water surface. When a laser beam is incident at a glancing angle on these surface waves, they act as a reflection grating, producing a well-defined diffraction pattern.

Surface tension waves are damped (their amplitude gradually decreases) as they propagate. This damping is due to the viscosity of the liquid, a property where adjacent layers of a liquid oppose relative motion between them.

Objective

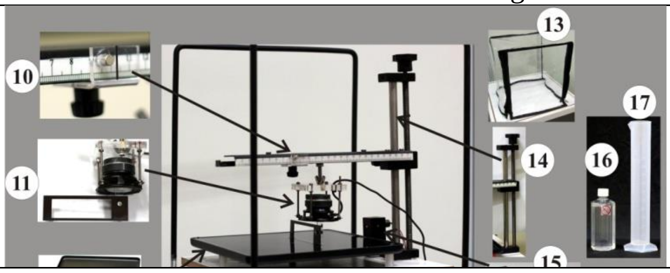
To use diffraction from surface tension waves on water to determine surface tension and viscosity of the given water sample.

List of apparatus



[1]	Light meter (connected to light sensor assembly)
[2]	Light sensor assembly mounted on vernier caliper placed on a screen base
[3]	Tablet computer (used as sine wave generator)
[4]	Digital multimeter
[5]	Vibrator control box
[6]	Wooden platform
[7]	Track for moving light sensor assembly
[8]	DC regulated power supply
[9]	Hex key, measuring tape and plastic scale

Figure 1: Wooden platform unit



[10]	Scale and rider with vibrator position marker
[11]	Vibrator assembly
[12]	Water tray
[13]	Plastic cover
[14]	Assembly for adjusting vibrator height
[15]	Laser source 2 (Wavelength, λ_L = 635 nm, 1nm = 10^{-9} m)

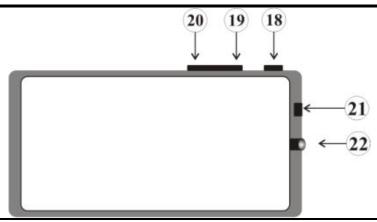
[1] *Shirish Pathare (HBCSE, Mumbai) and K G M Nair (CMI, Chennai) were the principal authors of this problem. The contributions of the Academic Committee, Academic Development Group and the International Board are gratefully acknowledged.*

	[16]	Water sample for the experiment
	[17]	500 ml measuring cylinder

Figure 2: Vibrator/laser source unit

Description of apparatus

a) Tablet computer as sine wave generator



[18]: Power Switch
[19]: Volume up
[20]: Volume down
[21]: Charging port
[22]: Socket for Audio connector pin of cable coming from vibrator control box[5]

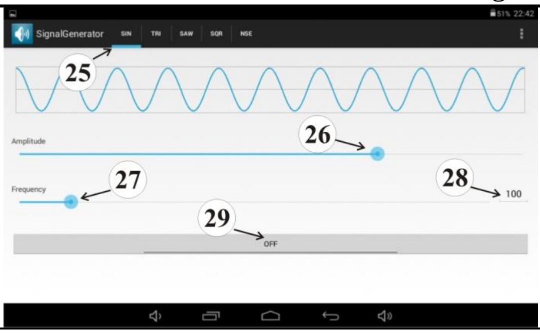
Figure 3: Switches and sockets of the tablet

Note
- Keep the tablet always charging.
- Gently press the power switch *once* to display initial screen.
- Keep the output volume at maximum using the “Volume up” button[19].

Touch and slide the icon[23] to unlock

Tap the icon [24] to start the sine wave generator

Figure 4: Initial screens of the tablet



[25]: Waveform selector (always keep at “SIN”)
[26]: Amplitude slider
[27]: Frequency slider
[28]: Frequency value field [Hz]
[29]: Application status indicator/switch “OFF” - the sine wave generator is OFF “ON” - the sine wave generator is ON

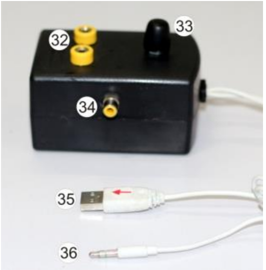
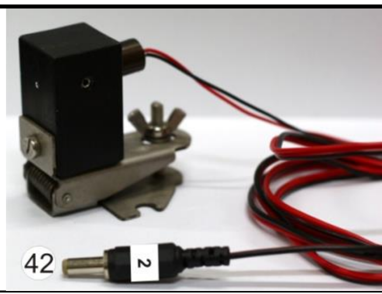
Figure 5: The sine wave generator application

To vary frequency
- Tap frequency value field [28] (Fig. 5) to reveal number pad
- Use backspace button [30] to erase frequency value
- Enter the required frequency, and press “Finished” button[31]

Figure 6: Screen showing number pad to enter frequency value

To vary amplitude
- Use amplitude slider [26] on tablet screen or the variable knob [33] on vibrator control box [5] to vary output amplitude.

b) Vibrator control box, digital multimeter, DC regulated power supply and their connections

		
[32]: Sockets to connect cables from multimeter	[37]: Vibrator strip	Figure 10: Laser source 2 [15] (mounted on a metal block) with connector [42]
[33]: Knob for varying amplitude of the sine wave	[38]: Pin of the cable from vibrator assembly	
[34]: Socket for pin of the cable from the vibrator assembly	Figure 8: Vibrator assembly[11]	
[35]: USB pin to be connected to DC regulated power supply	 [39]: AC/DC selector switch	[43]: Intensity switch (keep on "High" position)
[36]: Audio pin to be connected to the tablet	[40]: Range selector knob	[44]: USB socket for USB pin from vibrator control box
	[41]: Input sockets	[45]: Socket for connector from laser source 2
Figure 7: Vibrator control box[5]	Figure 9: Digital multimeter[4]	Figure 11: DC regulated power supply[8]

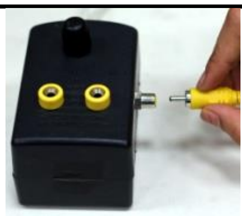
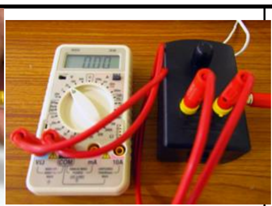
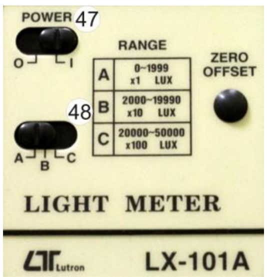
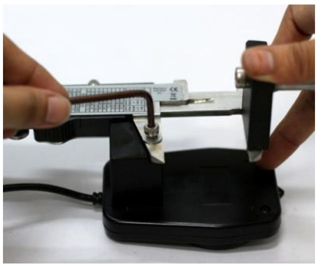
			
[36]→[22]	[38]→[34]	[41]↔[32]	[35]→[44] and [42]→[45]

Figure 12: Connections between tablet, vibrator control box and DC regulated power supply

c) Light sensor assembly and light meter

			
[46]: Circular aperture on the light sensor [47]: Power switch of the light meter [48]: A, B, C – Sensitivity ranges of the light meter		One jaw of the vernier caliper fits into a slot at the back of the light sensor	Tighten the screw using the hex key
Figure 13: Light sensor assembly and light meter		Figure 14: Attaching light sensor assembly	

Initial Adjustments

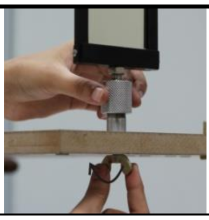
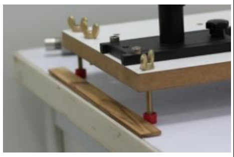
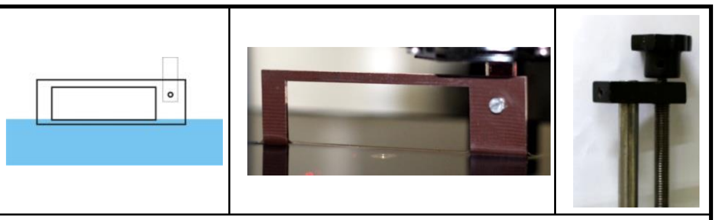
		

Figure 15: Removing the right reflector	Figure 16: Base screws touching the wooden strip	Figure 17: Correct position of the vibrator strip and black knob for height adjustment

1. Disconnect the laser 1 connector and insert the laser 2 connector into the socket of the DC regulated power supply. Note: Laser 2 has been already adjusted for a specific angle of incidence. Do not touch the laser source!

2. Remove the right reflector used in E-I by turning the bolt under the wooden platform (Fig. 15).

3. Remove the screen used in E-I and insert the light sensor assembly into the screen base. Place the screen base between the guiding rails of the track.

4. Position the wooden platform [6] with its base screws touching the wooden strip attached to the working table (Fig. 16).

5. Raise the side flap of the plastic cover on the vibrator/laser source unit. Pour exactly 500 ml of the water sample into the tray [12] using the measuring cylinder [17].

6. Switch on the laser. Locate the reflected laser spot on the light sensor. As you move the light sensor assembly back and forth along the track, the laser spot should move vertically and not at an angle to the vertical. Minor lateral adjustment of the wooden platform and vertical movement of light sensor assembly will allow you to get the laser spot exactly on the aperture. The intensity shown by the light meter will be maximum, if the centre of the laser spot coincides with the centre of the aperture,.

7. The vibrator strip has already been arranged in the correct vertical position. Do NOT change the black knob of the height adjustment assembly [14] (Fig. 17).

8. The vibrator assembly can be moved back and forth horizontally. Vibrator position marker indicates the position of the assembly on the scale [10].

9. While recording data, keep the flap of the plastic cover lowered in order to protect the water surface from air currents.

Experiment

Part C: Measurement of angle θ between the laser beam and the water surface

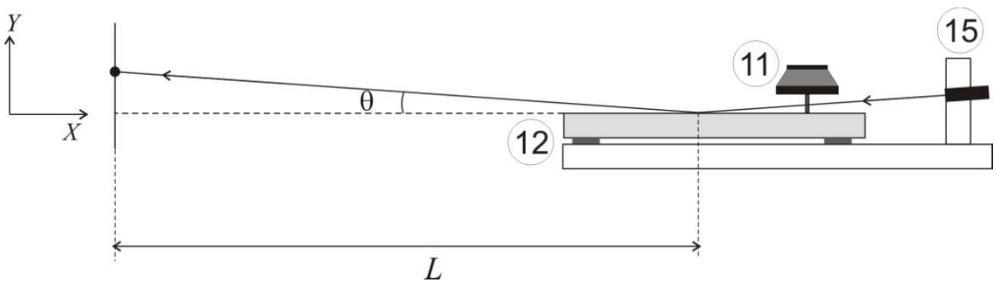


Figure 18: Measurement of angle θ

Tasks	Description	Marks
C1	Move the light sensor assembly in suitable steps along the track. Note down the X–displacement of the assembly and the corresponding Y-displacement of the laser spot. Record your readings in Table C1. (Select appropriate range in the light meter.)	**1.0**
C2	Plot a suitable graph (label it Graph C1) and determine the angle θ in degrees from its slope.	**0.6**

Part D: Determination of surface tension σ of the given water sample

From diffraction theory it can be shown that

$$k = \frac{2\pi}{\lambda_L} sin\theta\ sin\gamma \quad (1)$$

where, $k = {}^{2\pi}/_{\lambda_w}$ is the wave number of the surface tension waves,

λ_w and λ_L being the wavelengths of the surface tension waves and the laser respectively.

The angle γ is the angular distance between the central maximum and the first-order maximum (Fig. 19).

The vibration frequency (f) of the waves is related to the wave number k by

$$\omega = \sqrt{\frac{\sigma}{\rho} k^q} \quad (2)$$

where, $\omega = 2\pi f$, ρ is the density of the water and q is an integer.

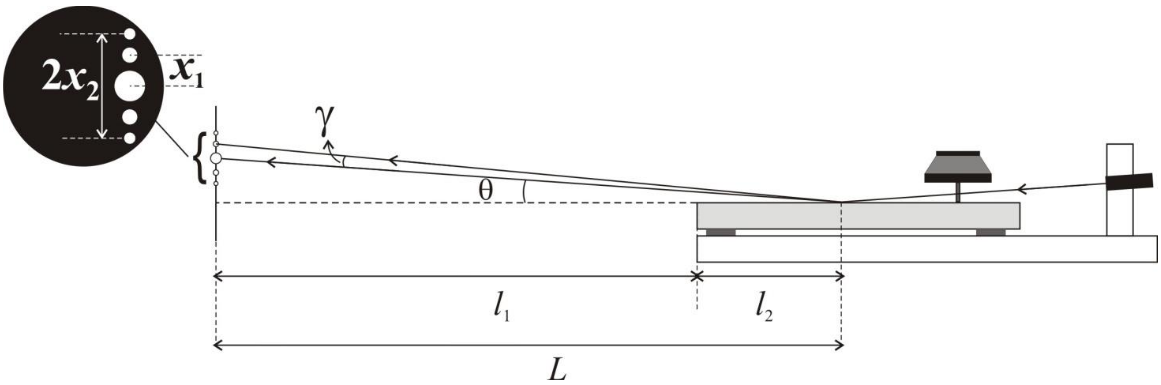


Figure 19: Schematic diagram of the apparatus

1. Fix the light sensor assembly (using the tightening bolt at the screen base) at the position shown in Fig. 1. Select the appropriate range on the light meter.

Task	Description	Marks
D1	Measure the length l_1 between the light sensor aperture and outer edge of the water tray. You will see a line where the laser strikes the water surface. The centre of this line is the point of incidence of the laser. Measure l_2, the distance of this point from the edge. Obtain L. Record it on your answersheet.	**0.3**

2. Set the vibrator position marker at 7.0 cm mark on the horizontal scale [10].
3. Set the sine wave frequency to 60 Hz and adjust its amplitude such that the first- and second-order maxima of the diffraction pattern are clearly visible (Fig. 19 inset).

Tasks	Description	Marks
D2	Measure the distance between the second-order maximum above and below the central maximum. Hence calculate x_1. Record your observations in Table D1. Repeat this by increasing the frequencies in appropriate steps.	**2.8**
D3	Identify the appropriate variables for a suitable graph whose slope would give the value of q. Enter the variable values in Table D2. Plot the graph to find q (label it Graph D1). Write down equation 2 with the appropriate integer value of q.	**0.9**
D4	From the equation 2, identify the appropriate variables for a suitable graph whose slope would give the value of σ. Enter the variable values in Table D3. Plot the graph to determine σ (label it Graph D2). (ρ =1000 kg.m^{-3}).	**1.2**

Part E: Determination of the attenuation constant, δ and the viscosity of the liquid, η

The surface tension waves are damped due to the viscosity of water. The wave amplitude, h, decreases exponentially with the distance, s, measured from the vibrator,

$$h = h_0 e^{-\delta s} \tag{3}$$

where, h_0 is the amplitude at the vibrator position and δ is the attenuation constant.

Experimentally, amplitude h_0 can be related to the voltage (V_{rms}) applied to the vibrator assembly as,

$$h_0 \propto (V_{rms})^{0.4} \tag{4}$$

The attenuation constant is related to the viscosity of the liquid as

$$\delta = \frac{8}{3}\frac{\pi\eta f}{\sigma} \tag{5}$$

where, η is the viscosity of the liquid.

1. Set the vibrator position marker at 8.0 cm.
2. Adjust the frequency to 100 Hz.
3. Adjust the light sensor using the vernier caliper such that the first-order maximum of the diffraction pattern falls on the aperture.
4. Adjust the amplitude of sine wave (V_{rms}) such that the reading in the light meter is 100 on range A. Note down V_{rms} corresponding to the light meter reading.
5. Move the vibrator away from the point of incidence of the laser in steps of 0.5 cm and adjust V_{rms} to get the light meter reading 100. Note down corresponding V_{rms}.

Tasks	Description	Marks
E1	Record your data for every step in Table E1.	**1.9**
E2	Plot a suitable graph (label it Graph E1) and determine the attenuation constant δ from its slope.	**1.0**
E3	Calculate the viscosity η of the given water sample.	**0.3**