July 09, 2015

General Instructions

- The theoretical examination lasts for 5 hours and is worth a total of 30 marks.
- You must not open the envelope containing the problems before the sound signal indicating the beginning of the competition.
- Dedicated IPhO Answer Sheets are provided for writing your answers. Enter the final answers into the appropriate boxes in the corresponding Answer Sheet (marked **A**). There are extra blank pages for carrying out detailed work/rough work (marked **D**). If you have written something on any sheet which you do not want to be graded, cross it out.
- Fill out all the entries in the header (Contestant Code, Q - T1,T2 or T3 and Page number).
- You may often be able to solve later parts of a problem without having solved the previous ones.
- You are not allowed to leave your working place without permission. If you need any assistance (malfunctioningcalculator, need to visit a restroom, etc), please draw the attention of the invigilator using one of the two cards (red card for help and green card for toilet).
- The beginning and end of the examination will be indicated by a sound signal. Also there will be sound signals every hour indicating the elapsed time. Additionally there will be a buzzer sound, fifteen minutes before the end of the examination (before the final sound signal).
- At the end of the examination you must stop writing immediately. Sort and numberyour Answer Sheets and detailed work sheets, put it in the envelope provided, and leave it on your table. You are not allowed to take any sheet of paper out of the examination area.
- Wait at your table till your envelope is collected. Once all envelopes are collected your student guide will escort you out of the examination area.
- A list of physical constants is given on the next page.

General Data Sheet

Acceleration due to gravity on Earth	g	$9.807\ \mathrm{m\ s^{-2}}$
Atmospheric pressure	P_{atm}	$1.013 \times 10^{5}\,\mathrm{Pa}$
Avogadro number	N_{A}	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Boltzmann Constant	k_{B}	$1.381 \times 10^{-23}\ \mathrm{J\ K^{-1}}$
Binding energy of hydrogen atom	$-$	$13.606\ \mathrm{eV}$
Magnitude of electron charge	e	$1.602 \times 10^{-19}\ \mathrm{C}$
Mass of the electron	m_{e}	$9.109 \times 10^{-31}\ \mathrm{kg}$
Mass of the proton	m_{p}	$1.673 \times 10^{-27}\ \mathrm{kg}$
Mass of the neutron	m_{n}	$1.675 \times 10^{-27}\ \mathrm{kg}$
Permeability of free space	μ_0	$1.257 \times 10^{-6}\ \mathrm{H\ m^{-1}}$
Permittivity of free space	ϵ_0	$8.854 \times 10^{-12}\ \mathrm{F\ m^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\ \mathrm{J\ s}$
Speed of sound in air (at room temperature)	c_{s}	$3.403 \times 10^{2}\,\mathrm{m\ s^{-1}}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\ \mathrm{m\ s^{-1}}$
Stefan-Boltzmann constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\ m^{-2}K^{-1}}$
Universal constant of Gravitation	G	$6.674 \times 10^{-11}\,\mathrm{N\ m^{2}\ kg^{-2}}$
Universal gas constant	R	$8.315\ \mathrm{J\ mol^{-1}K^{-1}}$

Particles from the Sun **(Total Marks: 10)**

Photons from the surface of the Sun and neutrinos from its core can tell us about solar temperatures and also confirm that the Sun shines because of nuclear reactions.

Throughout this problem, take the mass of the Sun to be $M_\odot = 2.00 \times 10^{30}$ kg, its radius, $R_\odot = 7.00 \times 10^{8}$ m, its luminosity (radiation energy emitted per unit time), $L_\odot = 3.85 \times 10^{26}$ W, and the Earth-Sun distance, $d_\odot = 1.50 \times 10^{11}$ m.

Note:

(i) $\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} + \text{ constant}$

(ii) $\int x^2e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} + \text{ constant}$

(iii) $\int x^3e^{ax}dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right)e^{ax} + \text{ constant}$

A Radiation from the sun :

A1	Assume that the Sun radiates like a perfect blackbody. Use this fact to calculate the temperature, T_s, of the solar surface.	**0.3**

The spectrum of solar radiation can be approximated well by the Wien distribution law. Accordingly, the solar energy incident on any surface on the Earth per unit time per unit frequency interval, $u(\nu)$, is given by

$$u(\nu) = A\frac{R_\odot^2}{d_\odot^2}\frac{2\pi h}{c^2}\nu^3 \exp(-h\nu/k_\mathrm{B}T_\mathrm{s}),$$

where ν is the frequency and A is the area of the surface normal to the direction of the incident radiation.

Now, consider a solar cell which consists of a thin disc of semiconducting material of area, A, placed perpendicular to the direction of the Sun's rays.

A2	Using the Wien approximation, express the total radiated solar power, P_in, incident on the surface of the solar cell, in terms of A, $R_\odot$, $d_\odot$, T_s and the fundamental constants c, h, k_B.	**0.3**
A3	Express the number of photons, $n_\gamma(\nu)$, per unit time per unit frequency interval incident on the surface of the solar cell in terms of A, $R_\odot$, $d_\odot$, T_s, ν and the fundamental constants c, h, k_B.	**0.2**

The semiconducting material of the solar cell has a "band gap" of energy, E_g. We assume the following model. Every photon of energy $E \geq E_\mathrm{g}$ excites an electron across the band gap. This electron contributes an energy, E_g, as the useful output energy, and any extra energy is dissipated as heat (not converted to useful energy).

A4	Define $x_\mathrm{g} = h\nu_\mathrm{g}/k_\mathrm{B}T_\mathrm{s}$ where $E_\mathrm{g} = h\nu_\mathrm{g}$. Express the useful output power of the cell, P_out, in terms of x_g, A, $R_\odot$, $d_\odot$, T_s and the fundamental constants c, h, k_B.	**1.0**
A5	Express the efficiency, η, of this solar cell in terms of x_g.	**0.2**
A6	Make a qualitative sketch of η versus x_g. The values at $x_\mathrm{g} = 0$ and $x_\mathrm{g} \to \infty$ should be clearly shown. What is the slope of $\eta(x_\mathrm{g})$ at $x_\mathrm{g} = 0$ and $x_\mathrm{g} \to \infty$?	**1.0**
A7	Let x_0 be the value of x_g for which η is maximum. Obtain the cubic equation that gives x_0. Estimate the value of x_0 within an accuracy of ± 0.25. Hence calculate $\eta(x_0)$.	**1.0**
A8	The band gap of pure silicon is $E_\mathrm{g} = 1.11$ eV. Calculate the efficiency, η_Si, of a silicon solar cell using this value.	**0.2**

In the late nineteenth century, Kelvin and Helmholtz (KH) proposed a hypothesis to explain how the Sun shines. They postulated that starting as a very large cloud of matter of mass, $M_\odot$, and negligible density, the

Sun has been shrinking continuously. The shining of the Sun would then be due to the release of gravitational potential energy through this slow contraction.

A9	Let us assume that the density of matter is uniform inside the Sun. Find the total gravitational potential energy, Ω, of the Sun at present, in terms of G, $M_\odot$ and $R_\odot$.	**0.3**
A10	Estimate the maximum possible time, τ_{KH} (in years), for which the Sun could have been shining, according to the KH hypothesis. Assume that the luminosity of the Sun has been constant throughout this period.	**0.5**

The τ_{KH} calculated above does not match the age of the solar system estimated from studies of meteorites. This shows that the energy source of the Sun cannot be purely gravitational.

B Neutrinos from the Sun :

In 1938, Hans Bethe proposed that nuclear fusion of hydrogen into helium in the core of the Sun is the source of its energy. The net nuclear reaction is:

$$4\ {}^{1}\mathrm{H} \longrightarrow {}^{4}\mathrm{He} + 2\mathrm{e}^{+} + 2\nu_{\mathrm{e}}$$

The "electron neutrinos", ν_{e}, produced in this reaction may be taken to be massless. They escape the Sun and their detection on the Earth confirms the occurrence of nuclear reactions inside the Sun. Energy carried away by the neutrinos can be neglected in this problem.

B1	Calculate the flux density, Φ_ν, of the number of neutrinos arriving at the Earth, in units of $\mathrm{m}^{-2}\mathrm{s}^{-1}$. The energy released in the above reaction is $\Delta E = 4.0 \times 10^{-12}\mathrm{J}$. Assume that the energy radiated by the Sun is entirely due to this reaction.	**0.6**

Travelling from the core of the Sun to the Earth, some of the electron neutrinos, ν_e, are converted to other types of neutrinos, ν_{x}. The efficiency of the detector for detecting ν_{x} is 1/6 of its efficiency for detecting ν_{e}. If there is no neutrino conversion, we expect to detect an average of N_1 neutrinos in a year. However, due to the conversion, an average of N_2 neutrinos (ν_{e} and ν_{x} combined) are actually detected per year.

B2	In terms of N_1 and N_2, calculate what fraction, f, of ν_{e} is converted to ν_{x}.	**0.4**

In order to detect neutrinos, large detectors filled with water are constructed. Although the interactions of neutrinos with matter are very rare, occasionally they knock out electrons from water molecules in the detector. These energetic electrons move through water at high speeds, emitting electromagnetic radiation in the process. As long as the speed of such an electron is greater than the speed of light in water (refractive index, n), this radiation, called Cherenkov radiation, is emitted in the shape of a cone.

B3	Assume that an electron knocked out by a neutrino loses energy at a constant rate of α per unit time, while it travels through water. If this electron emits Cherenkov radiation for a time, Δt, determine the energy imparted to this electron (E_{imparted}) by the neutrino, in terms of α, Δt, n, m_{e} and c. (Assume the electron to be at rest before its interaction with the neutrino.)	**2.0**

The fusion of H into He inside the Sun takes place in several steps. Nucleus of ${}^{7}\mathrm{Be}$ (rest mass, m_{Be}) is produced in one of these intermediate steps. Subsequently, it can absorb an electron, producing a ${}^{7}\mathrm{Li}$ nucleus (rest mass, $m_{\mathrm{Li}} < m_{\mathrm{Be}}$) and emitting a ν_{e}. The corresponding nuclear reaction is:

$${}^{7}\mathrm{Be} + \mathrm{e}^{-} \longrightarrow {}^{7}\mathrm{Li} + \nu_{\mathrm{e}}$$

When a Be nucleus ($m_{\mathrm{Be}} = 11.65 \times 10^{-27}$ kg) is at rest and absorbs an electron also at rest, the emitted neutrino has energy $E_\nu = 1.44 \times 10^{-13}$ J. However, the Be nuclei are in random thermal motion due to the temperature T_{c} at the core of the Sun, and act as moving neutrino sources. As a result, the energy of emitted neutrinos fluctuates with a root mean square (rms) value ΔE_{rms}.

B4	If ΔE_{rms}=5.54 $\times 10^{-17}$J, calculate the rms speed of the Be nuclei, V_{Be}, and hence estimate T_{c}. (Hint: ΔE_{rms} depends on the rms value of the component of velocity along the line of sight).	**2.0**

The Extremum Principle (Total Marks: 10)

A The Extremum Principle in Mechanics

Consider a horizontal frictionless $x-y$ plane shown in Fig. 1. It is divided into two regions, I and II, by a line AB satisfying the equation $x = x_1$. The potential energy of a point particle of mass m in region I is $V = 0$ while it is $V = V_0$ in region II. The particle is sent from the origin O with speed v_1 along a line making an angle θ_1 with the x-axis. It reaches point P in region II traveling with speed v_2 along a line that makes an angle θ_2 with the x-axis. Ignore gravity and relativistic effects in this entire task T-2 (all parts).

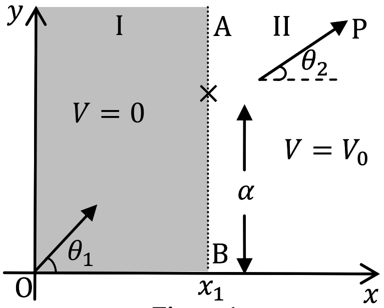


Figure 1

A1	Obtain an expression for v_2 in terms of m, v_1 and V_0.	**0.2**
A2	Express v_2 in terms of v_1, θ_1 and θ_2.	**0.3**

We define a quantity called action $A = m\int v(s)ds$, where ds is the infinitesimal length along the trajectory of a particle of mass m moving with speed $v(s)$. The integral is taken over the path. As an example, for a particle moving with constant speed v on a circular path of radius R, the action A for one revolution will be $2\pi mRv$. For a particle with constant energy E, it can be shown that of all the possible trajectories between two fixed points, the actual trajectory is the one on which A defined above is an extremum (minimum or maximum). Historically this is known as the Principle of Least Action (PLA).

A3	PLA implies that the trajectory of a particle moving between two fixed points in a region of constant potential will be a straight line. Let the two fixed points O and P in Fig. 1 have coordinates $(0,0)$ and (x_0, y_0) respectively and the boundary point where the particle transits from region I to region II have coordinates (x_1, α). Note that x_1 is fixed and the action depends on the coordinate α only. State the expression for the action $A(\alpha)$. Use PLA to obtain the relationship between v_1/v_2 and these coordinates.	**1.0**

B The Extremum Principle in Optics

A light ray travels from medium I to medium II with refractive indices n_1 and n_2 respectively. The two media are separated by a line parallel to the x-axis. The light ray makes an angle i_1 with the y-axis in medium I and i_2 in medium II (see Fig. 2). To obtain the trajectory of the ray, we make use of another extremum (minimum or maximum) principle known as Fermat's principle of least time.

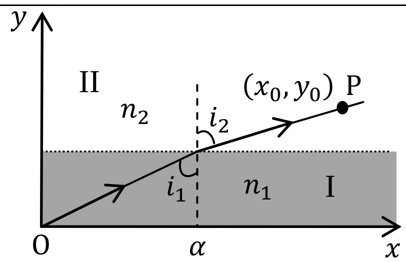


Figure 2

B1	The principle states that between two fixed points, a light ray moves along a path such that time taken between the two points is an extremum. Derive the relation between $\sin i_1$ and $\sin i_2$ on the basis of Fermat's principle.	**0.5**

Shown in Fig. 3 is a schematic sketch of the path of a laser beam incident horizontally on a solution of sugar in which the concentration of sugar decreases with height. As a consequence, the refractive index of the solution also decreases with height.

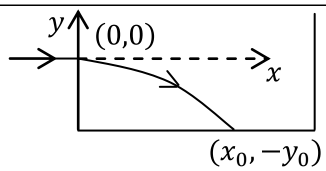
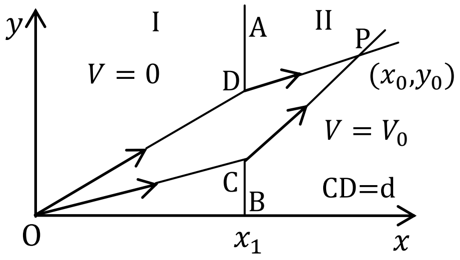


Figure 3: Tank of Sugar Solution

B2	Assume that the refractive index $n(y)$ depends only on y. Use the equation obtained in B1 to obtain the expression for the slope dy/dx of the beam's path in terms of refractive index n_0 at $y = 0$ and $n(y)$.	**1.5**
B3	The laser beam is directed horizontally from the origin (0,0) into the sugar solution at a height y_0 from the bottom of the tank as shown in figure 3. Take $n(y) = n_0 - ky$ where n_0 and k are positive constants. Obtain an expression for x in terms of y and related quantities for the actual trajectory of the laser beam. You may use: $\int \sec\theta d\theta = \ln(\sec\theta + \tan\theta) + \text{constant}$, where $\sec\theta = 1/\cos\theta$ or $\int \frac{dx}{\sqrt{x^2-1}} = \ln\left(x + \sqrt{x^2-1}\right) + \text{constant}$	**1.2**
B4	Obtain the value of x_0, the point where the beam meets the bottom of the tank. Take $y_0 = 10.0$ cm, $n_0 = 1.50$, $k = 0.050\ \text{cm}^{-1}$ (1 cm = 10^{-2} m).	**0.8**

C The Extremum Principle and the Wave Nature of Matter

We now explore the connection between the PLA and the wave nature of a moving particle. For this we assume that a particle moving from O to P can take all possible trajectories and we will seek a trajectory that depends on the constructive interference of de Broglie waves.

C1	As the particle moves along its trajectory by an infinitesimal distance Δs, relate the change $\Delta\varphi$ in the phase of its de Broglie wave to the change ΔA in the action and the Planck constant.	**0.6**
C2	Recall the problem from part A where the particle traverses from O to P (see Fig. 4). Let an opaque partition be placed at the boundary AB between the two regions. There is a small opening CD of width d in AB such that $d \ll (x_0 - x_1)$ and $d \ll x_1$. Consider two extreme paths OCP and ODP such that OCP lies on the classical trajectory discussed in part A. Obtain the phase difference $\Delta\varphi_{CD}$ between the two paths to first order.  **Figure 4**	**1.2**

D Matter Wave Interference

Consider an electron gun at O which directs a collimated beam of electrons to a narrow slit at F in the opaque partition A_1B_1 at $x = x_1$ such that OFP is a straight line. P is a point on the screen at $x = x_0$ (see Fig. 5). The speed in I is $v_1 = 2.0000 \times 10^7$ m s^{-1} and $\theta = 10.0000^\circ$. The potential in II is such that speed $v_2 = 1.9900 \times 10^7$ m s^{-1}. The distance $x_0 - x_1$ is 250.00 mm (1mm $= 10^{-3}$m). Ignore electron-electron interaction.

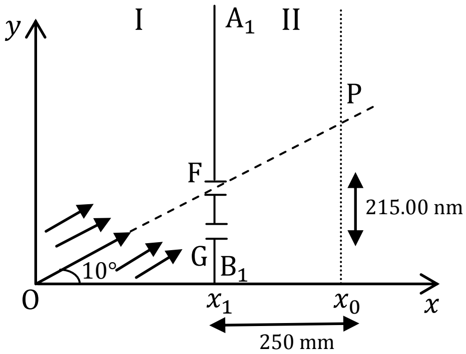


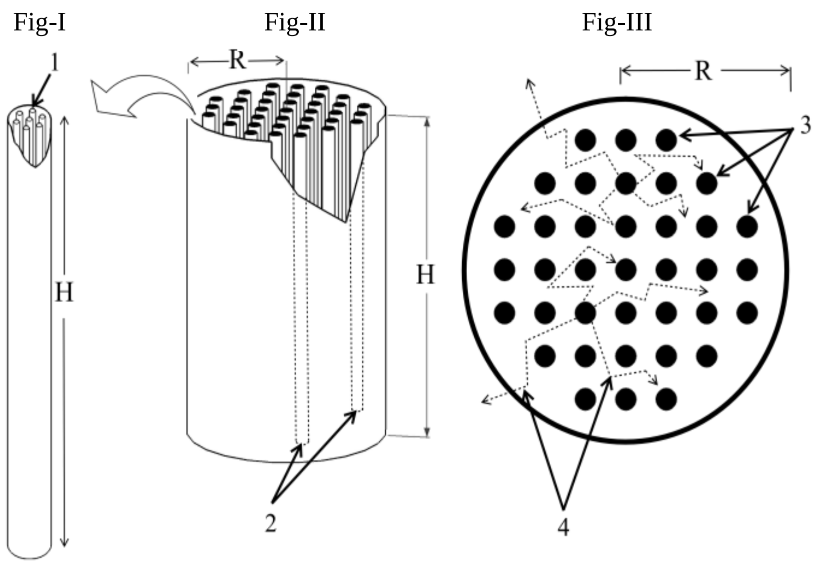
Figure 5

D1	If the electrons at O have been accelerated from rest, calculate the accelerating potential U_1.	**0.3**
D2	Another identical slit G is made in the partition A_1B_1 at a distance of 215.00 nm (1nm $= 10^{-9}$m) below slit F (Fig. 5). If the phase difference between de Broglie waves arriving at P through the slits F and G is $2\pi\beta$, calculate β.	**0.8**
D3	What is the smallest distance Δy from P at which null (zero) electron detection maybe expected on the screen? [Note: you may find the approximation $\sin(\theta + \Delta\theta) \approx \sin\theta + \Delta\theta\cos\theta$ useful]	**1.2**
D4	The beam has a square cross section of 500nm $\times$ 500nm and the setup is 2 m long. What should be the minimum flux density I_{min} (number of electrons per unit normal area per unit time) if, on an average, there is at least one electron in the setup at a given time?	**0.4**

The Design of a Nuclear Reactor **(Total Marks: 10)**

Uranium occurs in nature as UO_2 with only 0.720% of the uranium atoms being ^{235}U. Neutron induced fission occurs readily in ^{235}U with the emission of 2-3 fission neutrons having high kinetic energy. This fission probability will increase if the neutrons inducing fission have low kinetic energy. So by reducing the kinetic energy of the fission neutrons, one can induce a chain of fissions in other ^{235}U nuclei. This forms the basis of the power generating nuclear reactor (NR).

A typical NR consists of a cylindrical tank of height H and radius R filled with a material called moderator. Cylindrical tubes, called fuel channels, each containing a cluster of cylindrical fuel pins of natural UO_2 in solid form of height H, are kept axially in a square array. Fission neutrons, coming outward from a fuel channel, collide with the moderator, losing energy and reach the surrounding fuel channels with low enough energy to cause fission (Figs I-III). Heat generated from fission in the pin is transmitted to a coolant fluid flowing along its length. In the current problem we shall study some of the physics behind the (A) Fuel Pin, (B) Moderator and (C) NR of cylindrical geometry.



Schematic sketch of the Nuclear Reactor (NR)
Fig-I: Enlarged view of a fuel channel (1-*Fuel Pins*)
Fig-II: A view of the NR (2-*Fuel Channels*)
Fig-III: Top view of NR (3-*Square Arrangement of Fuel Channels* and 4-*Typical Neutron Paths*).
Only components relevant to the problem are shown (e.g. control rods and coolant are not shown).

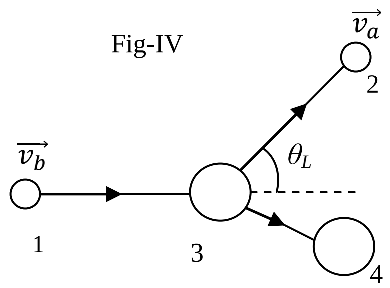
A Fuel Pin

Data for UO_2	1. Molecular weight M_w = 0.270 kg mol^{-1}	2. Density $\rho = 1.060\times10^4$ kg m^{-3}
	3. Melting point $T_m = 3.138\times10^3$ K	4. Thermal conductivity λ = 3.280 W m^{-1} K^{-1}

A1	Consider the following fission reaction of a stationary ^{235}U after it absorbs a neutron of negligible kinetic energy. $$^{235}\mathrm{U} + {}^{1}\mathrm{n} \rightarrow {}^{94}\mathrm{Zr} + {}^{140}\mathrm{Ce} + 2\,{}^{1}\mathrm{n} + \Delta E$$ Estimate ΔE (in MeV) the total fission energy released. The nuclear masses are: $m(^{235}\mathrm{U})$ = 235.044 u; $m(^{94}\mathrm{Zr})$ = 93.9063 u; $m(^{140}\mathrm{Ce})$ = 139.905 u; $m(^{1}\mathrm{n})$ = 1.00867 u and 1 u = 931.502 MeV c^{-2}. Ignore charge imbalance.	**0.8**
A2	Estimate N the number of ^{235}U atoms per unit volume in natural UO_2.	**0.5**
A3	Assume that the neutron flux density, $\varphi = 2.000\times10^{18}$ m^{-2} s^{-1} on the fuel is uniform. The fission cross-section (effective area of the target nucleus) of a ^{235}U nucleus is $\sigma_f = 5.400\times10^{-26}$ m^2. If 80.00% of the fission energy is available as heat, estimate Q (in W m^{-3}), the rate of heat production in the pin per unit volume. 1MeV = 1.602×10^{-13} J	**1.2**
A4	The steady-state temperature difference between the center (T_c) and the surface (T_s) of the pin can be expressed as $T_c - T_s = k\,F(Q,a,\lambda)$, where $k = 1/4$ is a dimensionless constant and a is the radius of the pin. Obtain $F(Q,a,\lambda)$ by dimensional analysis. Note that λ is the thermal conductivity of UO_2.	**0.5**
A5	The desired temperature of the coolant is 5.770×10^2 K. Estimate the upper limit a_u on the radius a of the pin.	**1.0**

B The Moderator

Consider the two dimensional elastic collision between a neutron of mass 1 u and a moderator atom of mass *A* u. Before collision all the moderator atoms are considered at rest in the laboratory frame (LF). Let $\overrightarrow{v_b}$ and $\overrightarrow{v_a}$ be the velocities of the neutron before and after collision respectively in the LF. Let $\overrightarrow{v_m}$ be the velocity of the center of mass (CM) frame relative to LF and θ be the neutron scattering angle in the CM frame. All the particles involved in collisions are moving at nonrelativistic speeds.

B1	The collision in LF is shown schematically, where θ_L is the scattering angle (Fig-IV). Sketch the collision schematically in CM frame. Label the particle velocities for 1, 2 and 3 in terms of $\overrightarrow{v_b}$, $\overrightarrow{v_a}$ and $\overrightarrow{v_m}$. Indicate the scattering angle θ. Fig-IV  *Collision in the Laboratory Frame* 1-*Neutron* before collision 2-*Neutron* after collision 3-*Moderator Atom* before collision 4-*Moderator Atom* after collision	**1.0**
B2	Obtain *v* and *V*, the speeds of the neutron and moderator atom in the CM frame after collision, in terms of *A* and v_b.	**1.0**
B3	Derive an expression for $G(\alpha, \theta) = E_a / E_b$, where E_b and E_a are the kinetic energies of the neutron, in the LF, before and after the collision respectively and $\alpha \equiv [(A-1)\ /\ (A+1)]^2$.	**1.0**
B4	Assume that the above expression holds for D_2O molecule. Calculate the maximum possible fractional energy loss $f_l \equiv \frac{E_b - E_a}{E_b}$ of the neutron for the D_2O (20 u) moderator.	**0.5**

C The Nuclear Reactor

To operate the NR at any constant neutron flux ψ (steady state), the leakage of neutrons has to be compensated by an excess production of neutrons in the reactor. For a reactor in cylindrical geometry the leakage rate is $k_1\,[(2.405/R)^2 + (\pi/H)^2]\,\psi$ and the excess production rate is $k_2\,\psi$. The constants k_1 and k_2 depend on the material properties of the NR.

C1	Consider a NR with $k_1 = 1.021\times10^{-2}$ m and $k_2 = 8.787\times10^{-3}$ m^{-1}. Noting that for a fixed volume the leakage rate is to be minimized for efficient fuel utilization, obtain the dimensions of the NR in the steady state.	**1.5**
C2	The fuel channels are in a square arrangement (Fig-III) with the nearest neighbour distance 0.286 m. The effective radius of a fuel channel (if it were solid) is 3.617×10^{-2} m. Estimate the number of fuel channels F_n in the reactor and the mass *M* of UO_2 required to operate the NR in steady state.	**1.0**

Experimental Examination

July 07, 2015

General Instructions

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- Wait at your tabletill your envelope is collected. Once all envelopes are collected your student guide will escort you out of the examination area.

Diffraction due to Helical Structure

(Total marks: 10)

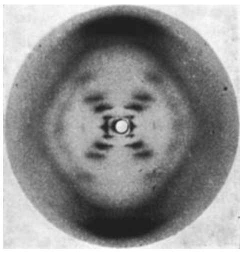


Figure 1: Photo 51

Introduction

The X-ray diffraction image of DNA (Fig. 1) taken in Rosalind Franklin's laboratory, famously known as "Photo 51", became the basis of the discovery of the double helical structure of DNA by Watson and Crick in 1952. This experiment will help you understand diffraction patterns due to helical structures using visible light.

Objective

To determine geometrical parameters of helical structures using diffraction.

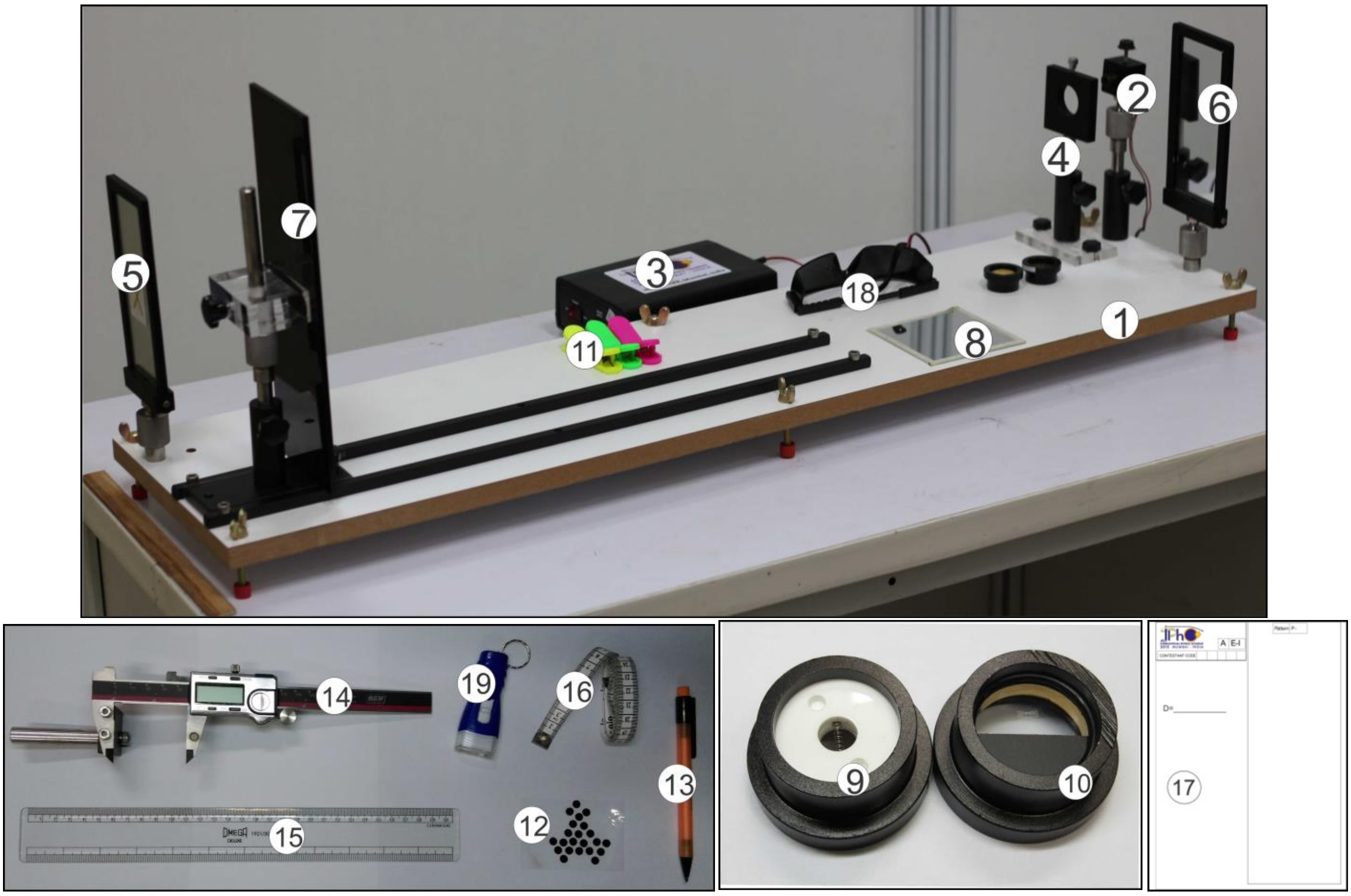


Figure 2: Apparatus for E-I

List of apparatus

[1]	Wooden platform	[11]	Plastic clips
[2]	Laser source with its mount and base	[12]	Circular black stickers
[3]	DC regulated power supply for the Laser source	[13]	Mechanical pencil
[4]	Sample holder with its base	[14]	Digital caliper with a mount
[5]	Left reflector (front coated mirror)	[15]	Plastic scale (30 cm)
[6]	Right reflector (front coated mirror)	[16]	Measuring tape (1.5 m)
[7]	Screen (10 cm x 30 cm) with its mount and base	[17]	Pattern marking sheets
[8]	Plane mirror (10 cm x 10 cm)	[18]	Laser safety goggles
[9]	Sample I (helical spring)	[19]	Battery operated flashlight
[10]	Sample II (double-helix-like pattern printed on glass plate)		

Note: Items [1], [3], [14], [15], [16] and [18] are also used in experiment E-II.

Description of apparatus

Wooden platform [1]: A pair of guiding rails, laser, reflectors, screen and sample mounts are rigidly fixed on it.

Laser source with its mount and base [2]: Laser source of wavelength $\lambda = 635$ nm (1 nm $= 10^{-9}$ m) is fixed in a metallic mount clamped to the base using a ball joint ([20] in Fig. 3) allowing the adjustment in X-Y-Z directions. The laser body can be rotated and clamped using the top lock-in screw. The beam focus can be adjusted by rotating the front lens cap (red arrow in Fig. 3) to obtain a clear and sharp diffraction pattern.

DC regulated power supply [3]: The front panel has an intensity switch (high/low), socket for laser source connector and three USB sockets. The back panel has power switch and mains power socket (inset of Fig. 4).

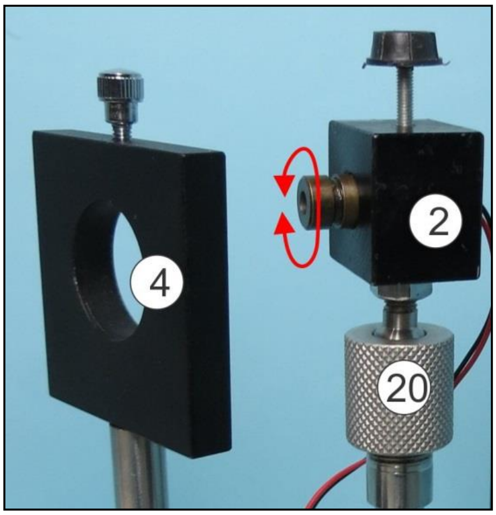


Figure 3: Laser source and sample holder. [20] Ball joint.

Figure 4: DC regulated power supply

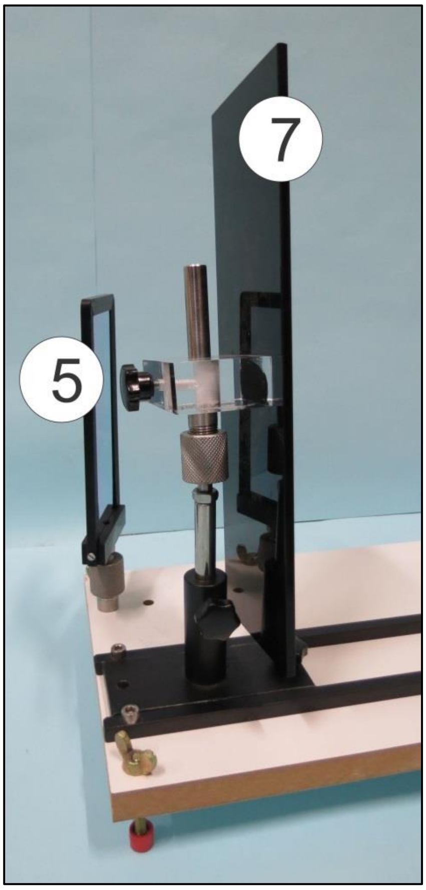


Figure 5: Left reflector and screen

Sample holder with its base [4]: Use the top locking screw to affix the samples in it (Fig. 3). The sample holder can be adjusted horizontally, vertically and rotated.

Left reflector [5]: This reflector is fixed to the platform (Fig. 5). Do not use the side marked X.

Right reflector [6]: This reflector is fixed to the platform and is removable (It will be removed in experiment E-II). Do not use the side marked X.

Screen with its mount [7]: The screen is mounted on ball joint and a base allowing rotational adjustments in all directions (Fig 5). The screen can be fixed as shown in Fig. 2 or Fig. 6 as required.

Sample I [9]: A helical spring fixed on a circular mount using white acrylic plates.

Sample II [10]: A double-helix-like pattern printed on a glass plate which is fixed on a circular mount.

Digital caliper with a mount [14]: Digital caliper is fixed to a mount (the mount is required in E-II). It has an On/Off switch, a switch to reset the reading to zero, a mm/inch selector (keep on mm), a locking screw and a knob for moving the right jaw. The digital caliper can be used to make measurements on pattern marking sheets.

Pattern marking sheets [17]: The given pattern marking sheets can be folded in half and clipped onto the screen using the plastic clips. Ensure that you mark the diffraction pattern within the rectangular box.

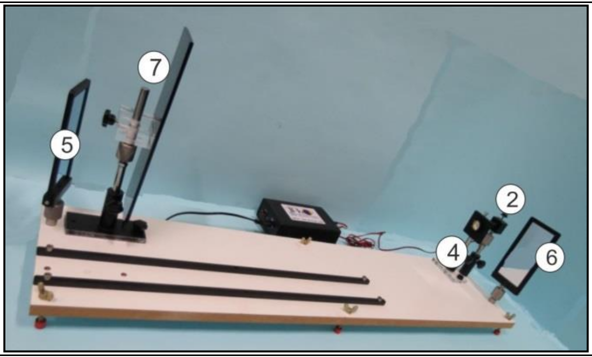


Figure 6: Alternate position of screen compared to that shown in Fig. 2

Theory

A laser beam of wavelength λ, falling normally on a cylindrical wire of diameter a, is diffracted in the direction perpendicular to the wire. The resulting intensity pattern as observed on a screen is shown in Fig. 7.

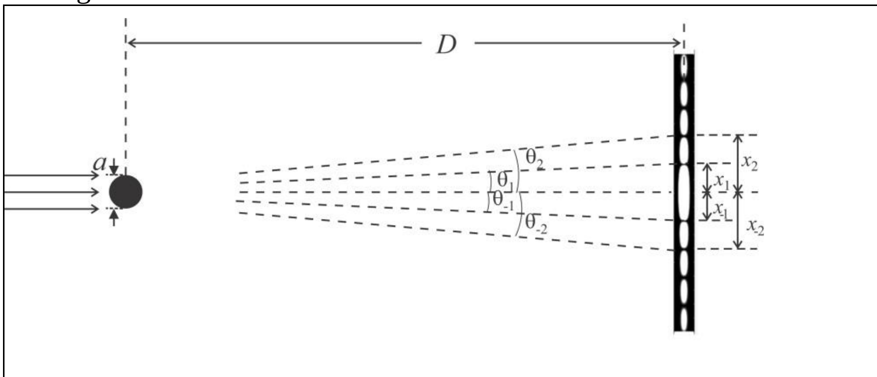


Figure 7: Schematic of the diffraction pattern due to single cylindrical wire of diameter a.

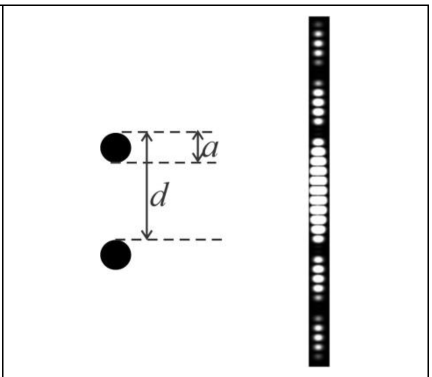


Figure 8: Schematic of diffraction pattern due to two cylindrical wires

The intensity distribution as a function of angle θ with the incident direction is given by

$$I(\theta) = I(0)\left[\frac{\sin\beta}{\beta}\right]^2 \qquad \text{where } \beta = \frac{\pi\, a \sin\theta}{\lambda}$$

The central spot is bright and for other angles, when $\sin\beta$ ($\beta \neq 0$) is zero, the intensity vanishes. Thus the intensity distribution has n^{th}minimum at the angle θ_n, given by

$$\sin\theta_n = \pm n\frac{\lambda}{a} \qquad n = 1, 2, 3, 4, 5 \ldots.$$

Here $\pm$ refers to both sides of the central spot ($\theta = 0$).

The diffraction pattern due to two parallel identical wires kept at a distance d from each other (Fig. 8) is a combination of two patterns (diffraction due to a single wire and interference due to two wires). The resultant intensity distribution is given by,

$$I(\theta) = I(0)\cos^2\delta\left[\frac{\sin\beta}{\beta}\right]^2$$

where $\delta = \frac{\pi\, d \sin\theta}{\lambda}$ and $\beta = \frac{\pi\, a \sin\theta}{\lambda}$

For a screen placed at a large distance D from the wire, the positions of the minima on the screen are observed at $x_{\pm n} = \pm n\frac{\lambda D}{a}$ due to diffraction and at $x_{\pm m} = \pm\left(m - \frac{1}{2}\right)\frac{\lambda D}{d}$ due to interference (where $m, n = 1, 2, 3, 4, 5 \ldots$). Similarly for a set of four identical wires (Fig. 9), the net intensity distribution is a combination of diffraction from each wire and interference due to pairs of wires and hence depends on a, d and s. In other words, the combination of three different intensity patterns is observed.

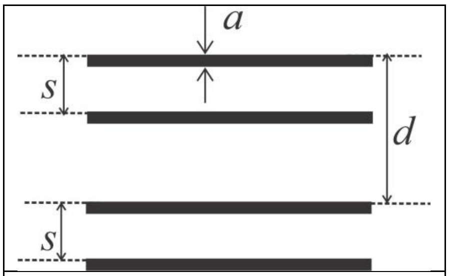


Figure 9: Set of four wires

Initial adjustments

1. Switch on the laser source and adjust both reflectors so that the laser spot falls on the screen.
2. Use the plastic scale and adjust the laser mount and reflectors such that the laser beam is parallel to the wooden platform.
3. Make sure that the laser spot falls near the centre of the screen.
4. Switch off the laser source. Clamp the pattern marking sheet on the screen.
5. Clamp the given plane mirror on the screen using plastic clips and switch on the laser again.
6. Adjust the screen so that the laser beam retraces its path back to the laser source. Remove the mirror once your alignment is completed.
7. Lights in the cubicle may be switched on/off as required.

Experiment

Part A: Determination of geometrical parameters of a helical spring

Sample I is a helical spring of radius R and pitch P made of a wire of uniform thickness a_1 as shown in Fig. 10(a). When viewed at normal incidence its projection is equivalent to two sets of parallel wires of the same thickness separated by distance d_1 and angle $2\alpha_1$ between them (Fig. 10(b)).

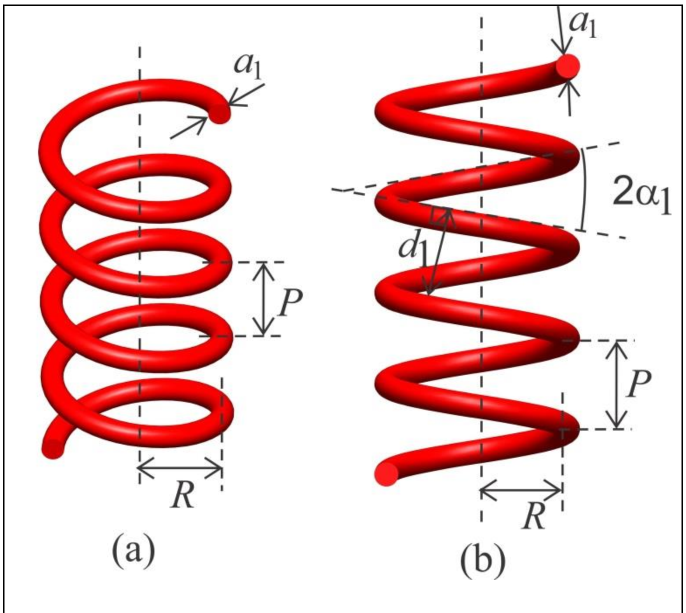


Figure 10: (a) Typical view of helical spring (b) Schematic diagram when viewed at normal incidence

- Mount sample I in the sample holder ensuring that the spring is vertical.
- Obtain a clear and sharp X-shaped diffraction pattern on the pattern marking sheet.

- For this you may adjust
  - laser beam focus (rotate lens cap)
  - beam orientation (rotate the laser body so that only two turns of the spring are illuminated)
  - laser intensity (high/low switch on power supply)
  - ambient light (by switching on or off cubicle light)

If the central maximum is very bright, you may stick circular black stickers on the pattern marking sheet to reduce scattering.

Tasks	Description	Marks
A1	Mark the appropriate positions (using given pencil [13]) of the intensity minima to determine a_1 and d_1 on the both sides of the central spot on the pattern-marking sheet. Please label your pattern-marking sheets as P-1, P-2 etc.	**0.7**
A2	Measure the appropriate distances using digital calipers and record them in Table A1 for determining a_1.	**0.5**
A3	Plot a suitable graph, label it Graph A1 and from the slope, determine a_1.	**0.7**
A4	Measure the appropriate distances and record them in Table A2 for determining d_1.	**0.8**
A5	Plot a suitable graph, label it Graph A2 and from the slope, determine d_1.	**0.6**
A6	From the X-shaped pattern, determine the angle α_1.	**0.2**
A7	Express P in terms of d_1 and α_1 and calculate P.	**0.2**
A8	Express R in terms of P and α_1 and calculate R (neglect a_1).	**0.2**

Part B: Determination of geometrical parameters of double-helix-like pattern

Figure 11(a) shows two turns of a double helix. Fig. 11(b) is a two-dimensional projection of this double helix when viewed at normal incidence. Each helix of thickness a_2 has an angle $2\alpha_2$ and perpendicular distance d_2 between turns. The separation between two helices is s. Sample II is a double-helix-like pattern printed on glass plate (Fig. 12), whose diffraction pattern is similar to that of a double helix. In this part, you will determine the geometrical parameters of sample II.

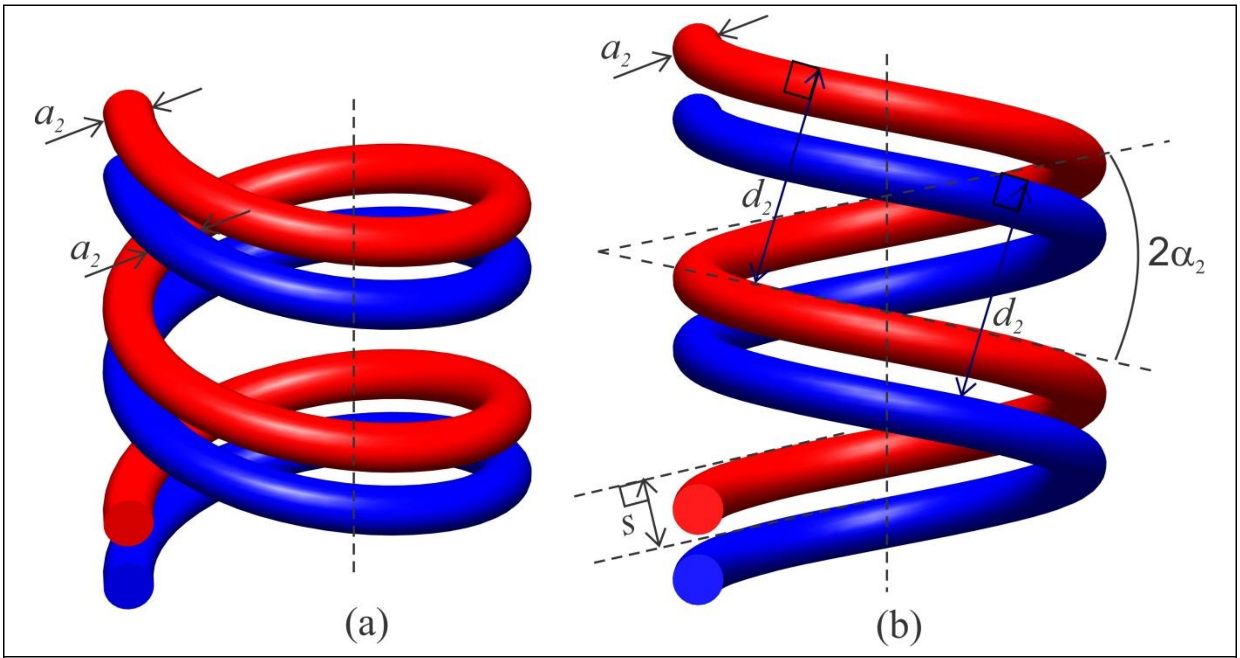


Figure 11: (a) Typical view of double-helical spring (b) Its schematic diagram when viewed at normal incidence.

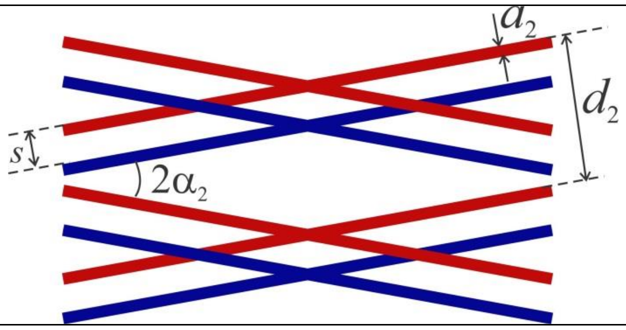


Figure 12: Double-helix-like pattern of sample II

- Mount the sample II in sample holder.
- Attach a new pattern-marking sheet on the screen.
- Obtain clear and sharp X-shaped diffraction pattern on the screen.

Tasks	**Description**	**Marks**
B1	Mark the appropriate positions of the minima on either side of the central spot to determine a_2, s and d_2. You can use more than one pattern marking sheets.	**1.1**
B2	Measure the appropriate distances and record them in Table B1 for determining a_2	**0.5**
B3	Plot suitable graph, label it Graph B1 and from the slope, determine a_2.	**0.5**
B4	Measure the appropriate distances and record them in Table B2 for determining s.	**1.2**
B5	Plot suitable graph, label it Graph B2 and from the slope determine s.	**0.5**
B6	Measure the appropriate distances and record them in Table B3 for determining d_2	**1.6**
B7	Plot suitable graph, label it Graph B3 and from the slope, determine d_2.	**0.5**
B8	From the X-shaped pattern, determine the angle α_2.	**0.2**

Diffraction due to surface tension waves on water

Introduction

Formation and propagation of waves on a liquid surface are important and well-studied phenomena. For such waves, the restoring force on the oscillating liquid is partly due to gravity and partly due to surface tension. For wavelengths much smaller than a critical wavelength, λ_c, the effect of gravity is negligible and only surface tension effects need be considered ($\lambda_c = 2\pi\sqrt{\frac{\sigma}{\rho g}}$, where σ is the surface tension, ρ is the density of the liquid and g is the acceleration due to gravity).

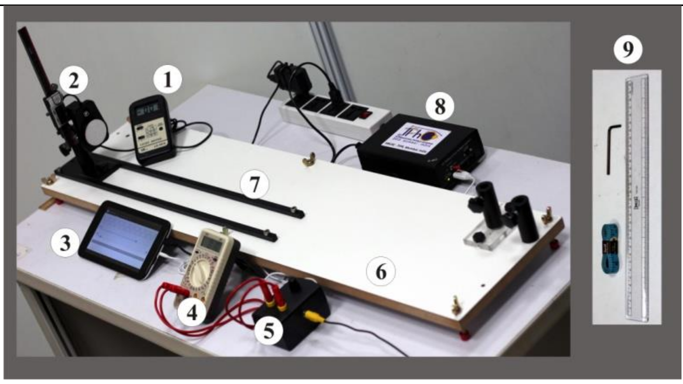
In this part, you will study surface tension waves on the surface of a liquid, which have wavelengths smaller than λ_c. Surface tension is a property of liquids due to which the liquid surface behaves like a stretched membrane. When the liquid surface is disturbed, the disturbance propagates as a wave just as on a membrane. An electrically-driven vibrator is used to produce waves on the water surface. When a laser beam is incident at a glancing angle on these surface waves, they act as a reflection grating, producing a well-defined diffraction pattern.

Surface tension waves are damped (their amplitude gradually decreases) as they propagate. This damping is due to the viscosity of the liquid, a property where adjacent layers of a liquid oppose relative motion between them.

Objective

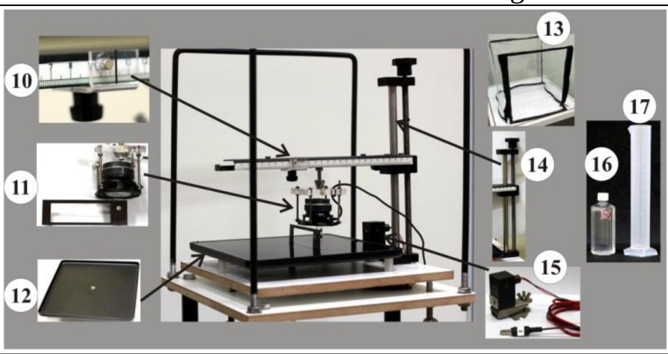
To use diffraction from surface tension waves on water to determine surface tension and viscosity of the given water sample.

List of apparatus



[1]	Light meter (connected to light sensor assembly)
[2]	Light sensor assembly mounted on vernier caliper placed on a screen base
[3]	Tablet computer (used as sine wave generator)
[4]	Digital multimeter
[5]	Vibrator control box
[6]	Wooden platform
[7]	Track for moving light sensor assembly
[8]	DC regulated power supply
[9]	Hex key, measuring tape and plastic scale

Figure 1: Wooden platform unit



[10]	Scale and rider with vibrator position marker
[11]	Vibrator assembly
[12]	Water tray
[13]	Plastic cover
[14]	Assembly for adjusting vibrator height
[15]	Laser source 2 (Wavelength, λ_L = 635 nm, 1nm = 10^{-9} m)
[16]	Water sample for the experiment
[17]	500 ml measuring cylinder

Figure 2: Vibrator/laser source unit

Description of apparatus

a) Tablet computer as sine wave generator

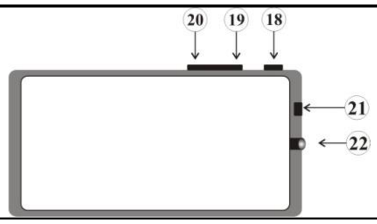
	[18]: Power Switch
	[19]: Volume up
	[20]: Volume down
	[21]: Charging port
	[22]: Socket for Audio connector pin of cable coming from vibrator control box[5]

Figure 3: Switches and sockets of the tablet

Note
- Keep the tablet always charging.
- Gently press the power switch *once* to display initial screen.
- Keep the output volume at maximum using the "Volume up" button[19].

	 Touch and slide the icon[23] to unlock		 Tap the icon [24] to start the sine wave generator

Figure 4: Initial screens of the tablet

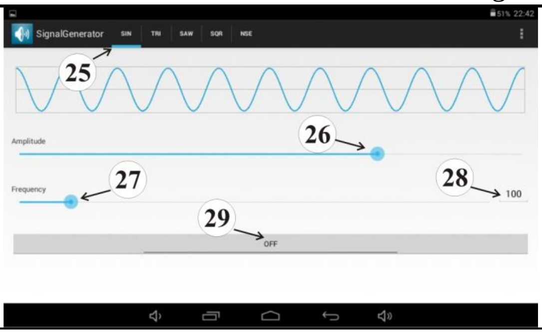
	[25]: Waveform selector (always keep at "SIN")
	[26]: Amplitude slider
	[27]: Frequency slider
	[28]: Frequency value field (Hz)
	[29]: Application status indicator/switch "OFF" - the sine wave generator is OFF "ON" - the sine wave generator is ON

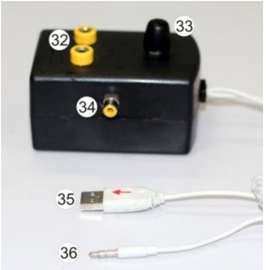
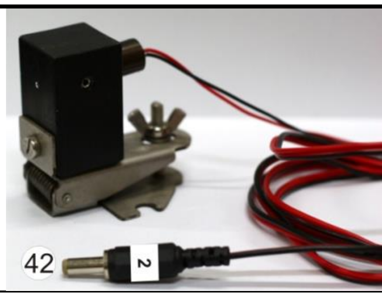
Figure 5: The sine wave generator application

	*To vary frequency* • Tap frequency value field [28] (Fig. 5) to reveal number pad • Use backspace button [30] to erase frequency value • Enter the required frequency, and press "Finished" button[31]

Figure 6: Screen showing number pad to enter frequency value

To vary amplitude
- Use amplitude slider [26] on tablet screen or the variable knob [33] on vibrator control box [5] to vary output amplitude.

b) Vibrator control box, digital multimeter, DC regulated power supply and their connections

		
[32]: Sockets to connect cables from multimeter	[37]: Vibrator strip	Figure 10: Laser source 2 [15] (mounted on a metal block) with connector [42]
[33]: Knob for varying amplitude of the sine wave	[38]: Pin of the cable from vibrator assembly	
[34]: Socket for pin of the cable from the vibrator assembly	Figure 8: Vibrator assembly[11]	
[35]: USB pin to be connected to DC regulated power supply	 [39]: AC/DC selector switch	[43]: Intensity switch (keep on "High" position)
[36]: Audio pin to be connected to the tablet	[40]: Range selector knob	[44]: USB socket for USB pin from vibrator control box
	[41]: Input sockets	[45]: Socket for connector from laser source 2
Figure 7: Vibrator control box[5]	Figure 9: Digital multimeter[4]	Figure 11: DC regulated power supply[8]

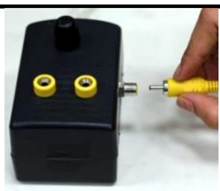
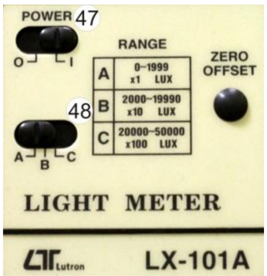
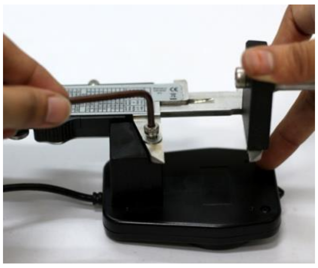
			
[36]→[22]	[38]→[34]	[41]↔[32]	[35]→[44] and [42]→[45]

Figure 12: Connections between tablet, vibrator control box and DC regulated power supply

c) Light sensor assembly and light meter

			
[46]: Circular aperture on the light sensor [47]: Power switch of the light meter [48]: A, B, C – Sensitivity ranges of the light meter		One jaw of the vernier caliper fits into a slot at the back of the light sensor	Tighten the screw using the hex key
Figure 13: Light sensor assembly and light meter		Figure 14: Attaching light sensor assembly	

Initial Adjustments

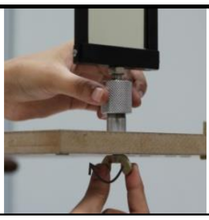
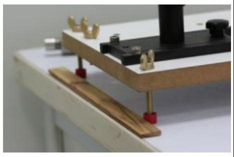
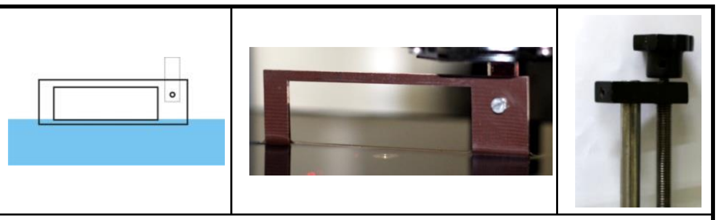
		

Figure 15: Removing the right reflector	Figure 16: Base screws touching the wooden strip	Figure 17: Correct position of the vibrator strip and black knob for height adjustment

1. Disconnect the laser 1 connector and insert the laser 2 connector into the socket of the DC regulated power supply. Note: Laser 2 has been already adjusted for a specific angle of incidence. Do not touch the laser source!

2. Remove the right reflector used in E-I by turning the bolt under the wooden platform (Fig. 15).

3. Remove the screen used in E-I and insert the light sensor assembly into the screen base. Place the screen base between the guiding rails of the track [7].

4. Position the wooden platform [6] with its base screws touching the wooden strip attached to the working table (Fig. 16).

5. Raise the side flap of the plastic cover on the vibrator/laser source unit. Pour exactly 500 ml of the water sample into the tray [12] using the measuring cylinder [17].

6. Switch on the laser. Locate the reflected laser spot on the light sensor. As you move the light sensor assembly back and forth along the track, the laser spot should move vertically and not at an angle to the vertical. Minor lateral adjustment of the wooden platform and vertical movement of light sensor assembly will allow you to get the laser spot exactly on the aperture. The intensity shown by the light meter will be maximum, if the centre of the laser spot coincides with the centre of the aperture,.

7. The vibrator strip has already been arranged in the correct vertical position. **Do NOT adjust** the black knob of the height adjustment assembly [14] (Fig. 17).

8. The vibrator assembly can be moved back and forth horizontally. Vibrator position marker indicates the position of the assembly on the scale [10].

9. While recording data, keep the flap of the plastic cover lowered in order to protect the water surface from air currents.

Experiment

Part C: Measurement of angle θ between the laser beam and the water surface

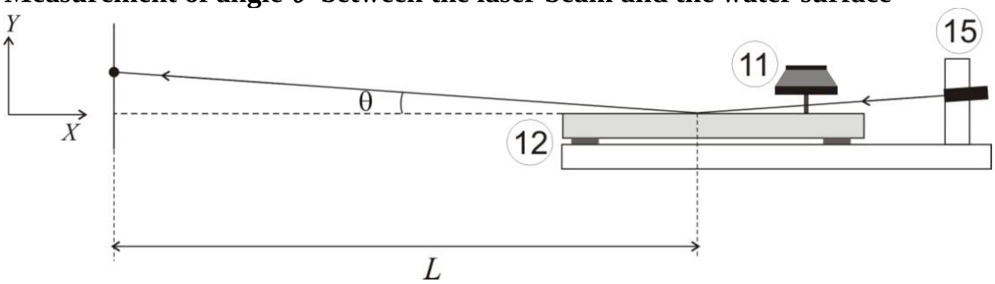


Figure 18: Measurement of angle θ

Tasks	Description	Marks
C1	Move the light sensor assembly in suitable steps along the track. Note down the X–displacement of the assembly and the corresponding Y-displacement of the laser spot. Record your readings in Table C1. (Select appropriate range in the light meter.)	**1.0**
C2	Plot a suitable graph (label it Graph C1) and determine the angle θ in degrees from its slope.	**0.6**

Part D: Determination of surface tension σ of the given water sample

From diffraction theory it can be shown that

$$k = \frac{2\pi}{\lambda_L} \sin\theta \sin\gamma \tag{1}$$

where, $k = {2\pi}/{\lambda_w}$ is the wave number of the surface tension waves,

λ_w and λ_L being the wavelengths of the surface tension waves and the laser respectively.

The angle γ is the angular distance between the central maximum and the first-order maximum (Fig. 19).

The vibration frequency (f) of the waves is related to the wave number k by

$$\omega = \sqrt{\frac{\sigma}{\rho} k^q} \tag{2}$$

where, $\omega = 2\pi f$, ρ is the density of the water and q is an integer.

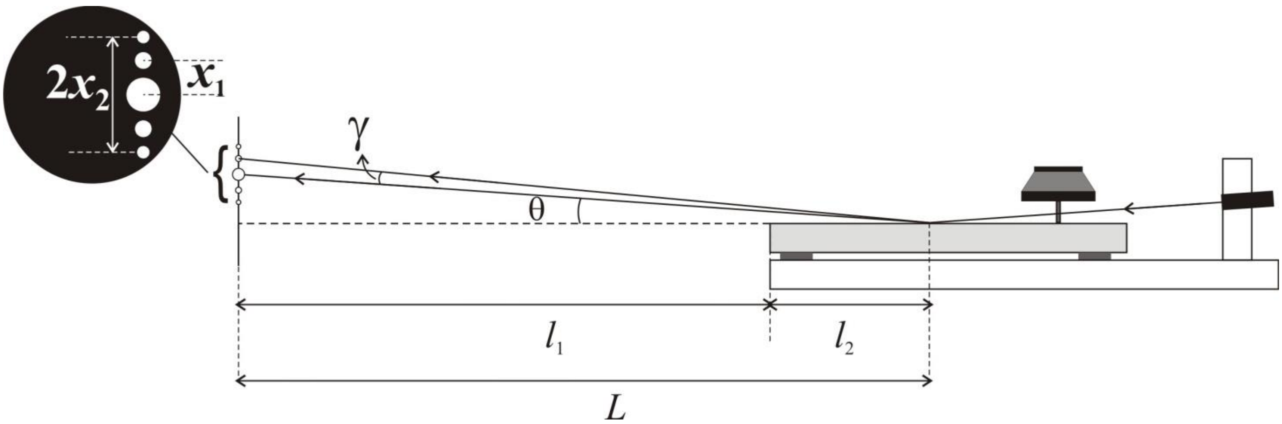


Figure 19: Schematic diagram of the apparatus

1. Fix the light sensor assembly [2] (using the tightening bolt at the screen base) at the end of the rails in the position shown in Fig. 1. Select the appropriate range on the light meter.

Task	Description	Marks
D1	Measure the length l_1 between the light sensor aperture and outer edge of the water tray. You will see a line where the laser strikes the water surface. The centre of this line is the point of incidence of the laser. Measure l_2, the distance of this point from the edge. Obtain L. Record it on your answersheet.	**0.3**

2. Set the vibrator position marker at 7.0 cm mark on the horizontal scale [10].
3. Set the sine wave frequency to 60 Hz and adjust its amplitude such that the first- and second-order maxima of the diffraction pattern are clearly visible (Fig. 19 inset).

Tasks	Description	Marks
D2	Measure the distance between the second-order maximum above and below the central maximum. Hence calculate x_1. Record your observations in Table D1. Repeat this by increasing the frequencies in appropriate steps.	**2.8**
D3	Identify the appropriate variables for a suitable graph whose slope would give the value of q. Enter the variable values in Table D2. Plot the graph to find q (label it Graph D1). Write down equation 2 with the appropriate integer value of q.	**0.9**
D4	From the equation 2, identify the appropriate variables for a suitable graph whose slope would give the value of σ. Enter the variable values in Table D3. Plot the graph to determine σ (label it Graph D2). (ρ =1000 kg.m^{-3}).	**1.2**

Part E: Determination of the attenuation constant, δ and the viscosity of the liquid, η

The surface tension waves are damped due to the viscosity of water. The wave amplitude, h, decreases exponentially with the distance, s, measured from the vibrator,

$$h = h_0 e^{-\delta s} \tag{3}$$

where, h_0 is the amplitude at the vibrator position and δ is the attenuation constant.

Experimentally, amplitude h_0 can be related to the voltage (V_{rms}) applied to the vibrator assembly as,

$$h_0 \propto (V_{rms})^{0.4} \tag{4}$$

The attenuation constant is related to the viscosity of the liquid as

$$\delta = \frac{8}{3}\frac{\pi\eta f}{\sigma} \tag{5}$$

where, η is the viscosity of the liquid.

1. Set the vibrator position marker at 8.0 cm.
2. Adjust the frequency to 100 Hz.
3. Adjust the light sensor using the vernier caliper such that the first-order maximum of the diffraction pattern falls on the aperture.
4. Adjust the amplitude of sine wave (V_{rms}) such that the reading in the light meter is 100 on range A. Note down V_{rms} corresponding to the light meter reading.
5. Move the vibrator away from the point of incidence of the laser in steps of 0.5 cm and adjust V_{rms} to get the light meter reading 100. Note down corresponding V_{rms}.

Tasks	Description	Marks
E1	Record your data for every step in Table E1.	**1.9**
E2	Plot a suitable graph (label it Graph E1) and determine the attenuation constant δ from its slope.	**1.0**
E3	Calculate the viscosity η of the given water sample.	**0.3**